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# THE BREAKING OF MULTIPLE BONDS AND THE PRIMARY DETONATING EXPLOSIVES

BY

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Detonation has been shown to be a violent manner of decomposition of certain compounds which are known as high explosives. This decomposition is transmitted through the material at rates varying from 2000 to 8000 meters per second depending on the type of material. The high explosives may be classified as primary and secondary detonating explosives (see Note No. 1).

This paper has to do with the sensitiveness of high explosives to detonation, and will seek to discover the answers by an examination of those materials which have sensitiveness of the highest order, that is, the primary detonating compounds or initiators (see Note No. 2). Sensitiveness is the "sine qua non" of explosives, and, therefore, a study of these extremely sensitive materials and their differences from other materials should serve to isolate the cause and nature of detonation. In this paper the writer will attempt also to supplement and complete the Breaking Theory of Detonation [1] and discuss the relation of breaking to chemical reaction in general (see Note No. 3).

The primary detonating explosives are almost always endothermic unsaturated compounds having multiple bonds. This

[1] JOUR. OF THE FRANKLIN INSTITUTE, July, Aug. and Sept., 1940.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

is their unique possession which sets them apart from other compounds. This feature of a chemical compound is probably the most potent force in the present day world of chemistry, for it makes possible the breaking of large molecules to smaller ones, and the combination of small molecules to make larger ones. Anyone familiar with petroleum and coke-oven by-products technology and the manufacture of plastics and synthetic rubber will readily agree that a thorough understanding of the nature of such bonds could be extremely valuable. In fact, it may be that chemical change itself, is dependent on the existence of distortions of the bond angles as a necessary prerequisite to breaking, which is the first phase of most reactions. Distortion of the bond angle is one of the characteristics of the multiple bond relation (see Note No. 4).

It would seem, then to be the logical procedure in our attempt to understand initial detonating materials and sensitivity in general, to see what can be discovered in the literature concerning the multiple bond.

Von Baeyer [2] was one of the pioneers in the investigation of this atomic relation. He contended that compounds having multiple bonds are under a real mechanical strain. The value of such early theories is verified by Mark [3] in speaking of free rotation around the single carbon-carbon bond. He says: "It is very remarkable that, in this respect, as well as in those of constant valence distance and deformability of valence angle, organic chemistry has evolved the correct answers merely through intuitive grasp of general relationships." Further remarks on the nature of the multiple bond relationship have been selected from the rather limited literature on the subject.

Mulliken [4] says: "The orbitals occupied by double-bond electrons are of two kinds, the first kind being essentially the same as occur in single bonds, the second being characteristic of double bonds. The outstanding feature of these special double bond orbitals is that they enforce a plane arrangement of all atoms directly connected to the double bond, or envolved in a system of conjugated or resonating double-bonds."

[2] *Ber.*, 18, 2278 (1885).

[3] "Physical Chem. of High Polymeric Systems," 1940, Interscience Pub. Co., N. Y.

[4] *J. Chem. Phys.*, 1939, Page 123.

On the same theme Slater [5] says: "Ethylene, with structure H H C : : C and Acetylene, with structure H : C ::: C : H are H H elementary examples of the double and triple bonds between carbons. The carbon-carbon distances are 1.34A and 1.22A in comparison with 1.54A for single bonds. This illustrates the general rule that internuclear distances are less with double bonds than with single, and still less with triple bonds. There is a feature in ethylene unlike the situation with the single bond in ethane, there is not the possibility of *free rotation* about the double bond. That is, the hydrogens all tend to lie in a plane, as our structural formula would indicate. This tendency is true in general with double bonds."

Houwink [6] also discusses the effect of the lack of free rotation as follows: "A third factor which may prevent the existence of brittleness in a substance is the presence of strong bonds next to weak ones. . . . In such a case it becomes understandable that the capacity for free rotation of certain groups in the chain molecule will work against the brittleness by presenting the possibility of yielding without breaking."

And Mark [7] says: "The loss of high axial symmetry merely indicates that the valence scaffolding of the two C atoms are now no longer free to rotate about the bond between the nuclei so that the double bond has a *certain rigidity*."

Henrich [8] states: "The resistance offered by the valences of carbon to deflection from their fixed positions around the atom shows itself in the form of a strain, and is comparable to the tension observed in the case of a bent metal spring." The following from Pauling [9] is, therefore, pertinent: "The result of the bond orbital treatment described above requires, however, that the carbon bond angle be close to the tetrahedral angle, since change from this value is associated with

[5] "Introduction to Chem. Physics," 1939, McGraw-Hill Book Co., Inc., N. Y.

[6] "Elasticity, Plasticity and the Structure of Matter," 1937, Cambridge Press, London.

[7] "Physical Chem. of High Polymeric Systems," 1940, Interscience Pub. Co., N. Y.

[8] "Theories of Organic Chemistry," 1922, John Wiley and Sons, N. Y.

[9] "The Nature of the Chemical Bond," 1939, Cornell Univ. Press, Ithaca, N. Y.

loss in bond strength of the carbon orbitals and hence decrease in stability of the system."

Two general statements by Lewis [10] on the nature of such linkages are given here: "Unsaturation connotes a set of properties, all of which indicate a looseness of structure and lack of stability." And: "Thus the various phenomena which indicate the breaking of a bond. . . . Such phenomena as the disruption of a molecule or the rearrangement of radicals, . . . , almost invariably occur in the neighborhood of such a seat of unsaturation. If not as an accurate scientific analogy, at least as a metaphor, we may liken the phenomenon to the result of a strain applied to some portion of a mechanical system of elastic bodies."

All of these authorities say about the same things in somewhat different words. The materials are under a strain; the bond angles are displaced from the normal position; there is absence of free rotation; there is chemical instability and "a certain rigidity."

These statements, then, all lead to the conclusion, that the materials with multiple bonds are, not only very reactive *chemically*, but are also very susceptible to physical or mechanical breaking, for as Evans [11] says: "One of the earliest, and certainly one of the most important consequences which followed the first structure analysis was a realization of the fact that no essential distinction exists between the 'chemical' forces responsible for binding together the atoms of the chemical molecule and the 'physical' forces responsible for the coherence of the solid as a whole, . . . ."

It would, therefore, seem proper and helpful, at this point, to present a sort of symposium of opinions as to the nature of breaking or rupture. That such a collection of opinions and experiences is necessary is affirmed by Southwell [12] who says: "We shall not attempt to adjudicate between these different theories, or to describe others which have been propounded in explanation of the elastic failure of materials. That the question is still open goes to emphasize our ignorance of the factors which determine the strength of materials."

[10] "Valence and the Structure of Atoms and Molecules," 1923, Reinhold Pub. Co., N. Y.

[11] "Crystal Chemistry," 1939, Cambridge Univ. Press,

[12] "Theory of Elasticity." 1936, The Clarenden Press, Oxford.

Bridgman [13] says: "For myself, I am exceedingly sceptical as to whether there is any such thing as a genuine criterion of rupture. For example, it is obvious enough that the manner of failure of hollow cylinders exposed to external pressure was entirely different for amorphous and crystalline Substances. I believe we have no right to expect any general criterion of rupture in view of the extremely varied structure of different sorts of material." The same writer [14] in another place says: "The phenomena of snapping or rupture already discussed may be importantly affected by rate of deformation. This is not merely an increased frequency in time, but the ruptures are separated by smaller angular displacements" (see Note No. 6). And still further on he says: "The phenomenon (rupture) is specific with the material."

From the point of view that the phenomenon *is* specific with the material the following references are given here.

Stillwell [15] says: "A sheet or thread made of short crystals in random orientation, however, is brittle because there is no reinforcement by interlocking of the crystals," and "A sheet of high grade gelatin is pliable and not easily broken, but a film formed from gelatin which has been broken down by hydrolysis (i.e., the molecules shortened) is relatively brittle."

Bancroft [16] defines some terms as follows: "A material is brittle when a slight permanent deformation involves sudden rupture and when impact produces shattering. This means that the yield point and the break-point are practically identical, as with cast iron, while the two are far apart in the case of a ductile material. A material is tough when the resistance to permanent deformation is marked, but permanent deformation does not involve rupture; and when the resistance to rupture on impact is marked. In other words brittleness is antithetical both to toughness and ductility. A material is plastic when permanent deformation involves no rupture and a break is self healing." And this very significant statement: "Sealing wax is brittle if the pressure is applied rapidly, but is plastic if the rate of deformation is slow."

[13] "The Physics of High Pressure," The Macmillan Co., N. Y., 1931.
[14] "The Journal of Geology," Vol. 44, No. 6, 1936.
[15] "Crystal Chemistry," 1938, McGraw-Hill Book Co., N. Y.
[16] "Applied Colloid Chemistry," 1932, McGraw-Hill Book Co., N. Y.

Houwink [17] states: "Because of the congruence of the potential troughs for a crystalline body, and the fact that the energy necessary to remove each atom from its trough is about the same, the yield value of such a body is rather sharp, and by comparison the yield value of an amorphous body is unsharp."

Now to return to a more detailed study of rupture. Bridgman [18] makes this statement: "Under normal conditions, before the solid breaks, every atom is in a stable force field, that is, the net force called into play when the atom is displaced from its position of equilibrium is in such a direction as to restore it to its initial position. When this condition fails to hold, the atom finds itself in an unstable force field, and will presently take up a new position of equilibrium at some distance from its original position. It seems plausible to suppose that every fracture must take its microscopic beginning from some such atomic instability. Whether the instability will then spread from microscopic to macroscopic dimensions will depend in part on large scale considerations, principally on the energy. In particular, the instability will tend to spread if in this way the potential of all the forces is diminished. It seems to me that any theory of rupture must envolve as an essential element a consideration of this instability factor, for certainly if there is instability there will be fracture and conversely, fracture is an instability." Bridgman's statement that fracture is an instability of course refers to the fact that the atoms must move after fracture occurs to re-establish equilibrium.

It will be noted that Lewis states that unsaturation indicates a lack of stability and the statement above emphasizes the fact that instability must precede rupture. The first is due to unsaturation; a inherent internal condition; the second to a change in external condition.

Rupture, incidently, produces another instability by creating a new surface of which Rideal says: [19] "The surface of a crystal is thus chemically unsaturated and the existence of

[17] "Elasticity, Plasticity and Structure of Matter," Cambridge Press, 1937, London.

[18] *Jour. of Applied Physics*, Vol. 9, No. 8, Aug. 1938.

[19] "Surface Chemistry," 1930, Cambridge Univ. Press, London.

a definite surface energy is due to the fact that the surface is unsaturated." And Davey [20] says: "Estimates of internal pressure and measurements on the surface tension of molten metals and salts at their freezing temperatures lead us to believe that these surface tension forces in crystals are very large. . . . The surface of a crystal must therefore be assumed to be under a heavy stress."

In this case, then, breaking has caused a part of the material to be under a heavy stress and to be more reactive chemically.

Again quoting from Bridgman [21] we have the following: "A complete theory of rupture would involve a formulation of all the conditions under which instabilities may arise. . . . The situation is further complicated by the probability that instabilities are almost always initiated at some imperfection in the structure, either internal or on the surface, . . . It is possible to see from certain very general considerations, however, that there are factors of instability present in any ordinary solid, even in the absence of imperfections in the structure."

A statement by Davey [22] seems pertinent at this point, he says: "Perfection in single crystals is so rare that we may confidently assume imperfection to be their normal state."

Houwink [23] in discussion of Locherstellen (imperfections) says: "The question may now arise; what would be the behavior of these substances if they had no such weak spots? It seems fairly obvious that they might be extremely strong but also extremely brittle. If a very high load were suddenly applied, they would perhaps disintegrate into separate atoms, which would be the utmost limit of rigid behavior" (see Note No. 5).

Experimental results supporting Houwink's theoretical deduction are found in Bridgman's [24] use of tungsten carbide

[20] "Study of Crystal Structure and Its Application," 1934, McGraw-Hill Book Co., N. Y.

[21] *Jour. of Applied Physics*, Vol. 9, No. 8, Aug. 1938.

[22] "Study of Crystal Structure and Its Application," 1934, McGraw-Hill Book Co., N. Y.

[23] "Elasticity, Plasticity and Structure of Matter," 1937, Cambridge Press, London.

[24] *Proc. of the Am. Acad. of Art and Sciences*, 74, No. 3, Oct. 1940.

(a extremely strong and brittle material) under very high pressures. Bridgman says: "Final rupture of a piston may be a most violent affair, when it is often reduced to a fine powder."

It might be said, then, that the absence of imperfection or the sudden-ness of the applied high load might reduce the importance of the imperfections to a negligible factor of influence on rupture. In any case the attempt to explain rupture by the presence of imperfections offers very little in the nature of a solution of the problem. The fact that imperfections are quite universally present prevents their use as a basis for differentiation between materials susceptible to rupture.

It seems to me that the outstanding and definite lessons to be derived from these opinions and experiences (See note no. 5A) are the following:

1. That the most rigid materials fracture most readily and with the least deformation.

2. That soft, flexible and plastic materials resist fracture.

3. That a high rate of energy application will often produce fracture when a low rate of application will not, and that fracture occurs with the high rate of energy application with a smaller deformation of the material before rupture (see Note No. 6).

4. When fracture occurs the atoms move to new positions. Ordinary fracture produces a new surface which is under a stress and is chemically unsaturated and chemically active.

5. That instability must precede rupture. It is the opinion of some that the instability originates at some imperfection (lockerstellen) in the structure. However, imperfection is a universal and normal state, whereas, distortion of the bond angle is known to be an instability, which is inherent with some compounds, but which can be produced in others by the application of outside forces.

Now we will turn aside to consider some fairly well known characteristics of explosives with principal attention to the type of energy required to cause their detonation, and the physical condition which makes them susceptible to detonation.

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Colvin [25] speaking of the nitroglycerine explosives in the early days of their manufacture says: "Ignition by flame or fuse was found to be totally inadequate for these new explosives. . . . Indeed, Nitroglycerine could be ignited and allowed to burn with the development of a flame without any fear of explosion; but a blow or slight percussion gave rise to an explosion of considerable intensity."

The solution for this problem, as we all know, was the discovery by Nobel of the effectiveness of mercury fulminate as an initiator of detonation. Some opinions regarding the cause of its effectiveness are given here. Wohler [26] attributed the effectiveness of mercury fulminate to the production of an enormous pressure. Nobel [27] says in his patent specifications: "The principal of its action consists invariably in the production of a very intense shock or pressure." Berthelot [28] agrees with the above that the initiating effect is due to the sudden development of enormous pressure.

Picric acid, the well known explosive, was discovered by Woulfe [29] in 1771 but it was not until 1873 that Sprengel found that it is "a powerful explosive when fired with a detonator." When this material is heated slowly it melts and decomposes without any indication of explosion. But a sudden blow or very rapid heating may cause detonation.

An example is given by Bernsthen [30] which shows the effect of physical state on sensitiveness: "The nitro derivatives (nitro Methane and ethane) are colorless liquids, . . . . Like the later they distil without decomposition, and occasionally explode when quickly heated. Nitro-methane boils at 99–101° C. Nitro-ethane boils at 113–114, burns with a bright flame, and the vapour does not explode even at a high temperature." And speaking of the sodium salts of these materials: "These sodium salts are *crystalline solids* and are highly explosive."

[25] "High Explosives," 1918, C. Lockwood and Sons, London.

[26] *Zeit. f. Ges. Schiess. u. Spreng.*, 1907, Vol. 2, pp. 181–203, 244–265.

[27] English Patent No. 1345, May 7, 1867.

[28] "Explosives and Their Power," 1892, J. Murray, London.

[29] Reports of Invest. Dept. of Int., Bur. of Mines, Ser. No. 2243, April, 1921.

[30] "Text-book of Organic Chemistry," Sudborough, D. Van Nostrand Co., 1912, N. Y.

Reese says:[31] "A frozen stick of dynamite is liable to explode when broken in two, probably from internal friction between the crystals."

An example in which there is resistance to breaking due to lack of rigidity is a material which has all of the chemical characteristics of a good initiator. This material is more endothermic than any of the known solid initiators. Its heat of formation is − 1260 Cal. per gram as compared with − 346 for lead azide, − 227 for mercury fulminate and − 691 for sulphur nitride. Marshall[32] says of this compound: "Cyanogen $C_2N_2$, for instance, has a heat of formation of − 68.5 Cal. per Mol. or − 132 Cal. per Kg. in the liquid state, yet it can only be detonated with difficulty and does not explode at all on heating."

Muraour[33] says: "In the manufacture of detonators with fulminate at extremely high loading pressures (above 700 Kg./Cm.$^2$) (approximately 9960 lbs. per Sq. in.) ignition by the flame of a Bickford fuse does not cause detonation, but causes simple combustion of the explosive. . . . Furthermore, the explosive so dead pressed is more susceptible to normal detonation if an explosive wave is started by the detonation, of a small amount of an initiating explosive, such as lead azide."

The above statement verifies the work of Taylor and Munroe[34] and Storm[35] who found that fulminate pressed at 10,500 lbs. per Sq. in. burned without bursting the detonator capsules.

The findings given in the two paragraphs above are to be contrasted with the results secured with the same compound when initiation was accomplished by the impact of a firing pin. Fulminate detonators pressed at 20,000 lbs. per Sq. in. functioned perfectly and those pressed at 30,000 lbs. per Sq. in. failed in only four trials out of a total of twenty-four.

[31] JOUR. OF FRANKLIN INSTITUTE, Vol. 198, 1924, Page 745.

[32] "Explosives," Vol. 2, 1917, Blakiston, Philadelphia.

[33] "Memorial de L'Artillerie Française," Vol. 18, No. 4, 1939, p. 895. Translation by P. Varrato.

[34] Reports of Investigation, Dept. of Int. Bur. of Mines, Ser. No. 2558, Dec. 1923.

[35] "Dead Pressed Explosives," Army Ordnance, 3–58 (1922).

Rinkenbach [36] says in reference to lead azide: "Furthermore it has been recognized and noted that the sensitivity is dependent upon the crystal size; this sensitivity being so great as to render the handling of the material extremely dangerous if the crystals exceeded certain sizes."

In describing the properties of cyanuric triazide Taylor and Rinkenbach [37] say: "If heated slowly, it may entirely decompose without detonation, but detonates quickly by flame or sudden heating. The small crystals are more sensitive to impact and friction than mercury fulminate and have detonated when being pressed in a detonator capsule. The large crystals, . . . have detonated when broken by a rubber policeman" (see Note No. 7).

The writer feels that the writers of the last sentence did not wish to convey the idea that the crystal broke and subsequently detonated, with a perceptible interval of time intervening. What happened, perhaps, could be stated in this way: A pressure was applied to the crystal which ordinarily would be expected to cause breaking but which actually caused it to detonate.

Bridgman [38] says: "The stability of every atom is determined by its position with respect to its neighbors; when its position relative to its neighbors is changed, as by the application of forces to the surface of the body, its stability will also change."

The possibility of such spontaneous detonations is suggested in Ward's [39] theoretical discussion of the crystal as follows: "An atom inside the crystal has less energy than when it is outside, since otherwise a crystal would fly apart." We know that the surfaces of all crystals are unsaturated, and therefore, the surface atoms have greater energy than the atoms in the interior when the crystal itself is a saturated compound. But the crystal of a unsaturated compound has internal unsaturation and therefore, the atoms of the interior may have nearly as much energy as those on the surface, so that any slight disturbance such as a slight stress applied to

[36] U. S. Patent No. 1,914,530, June 20, 1933.

[37] Reports of Invest. Dept. of Int. Bureau of Mines, Ser. 2513, Aug. 1923.

[38] *Jour. of Applied Physics*, Vol. 9, No. 8, 1938.

[39] "The Nature of Crystals," 1938, Blackie and Sons Ltd., London.

the crystal as a whole could cause the atoms of the interior to possess greater energy than the surface atoms and cause the crystal "to fly apart" or detonate.

Bridgman [40] has produced what he chooses to call mechanical detonations of many compounds by shearing under very high pressures. He has also produced detonation of celluloid by the use of high pressures only. He states that the detonations are of physical origin and are not due to chemical instability. To the writer this seems a chimerical distinction in view of reference 11 quoted on a preceding page and which states that no essential difference exists between the chemical and physical forces. In fact, one of the principal objects for the writing of this paper, is to contend that there is no distinction and to show how the acceptance of such a contention can explain many phenomena more precisely than any other theory.

Many explosives, such as cyanuric triazide, picric acid and T.N.T., melt at comparatively low temperatures. If such explosives depended on being heated to a high temperature for detonation, then detonation must need to be a very much slower process than is the fact. For heat would have to be added by conduction, a comparatively slow process. Each of these explosives would have to pass through their melting points and have heat supplied for the heat of fusion, which in the case of T.N.T. is 22.3 Cal. per gram. The crystal size of these explosives have a marked effect on their sensitiveness. If, therefore, such materials were heated to their "so-called" explosion temperature their crystals would be non-existent and could have no effect on the sensitiveness.

Now, considering the case of the so-called mechanical detonations produced by Bridgman and the detonation of cyanuric triazide which was caused by subjecting the large crystals to pressure, it must be conceded that these materials were made sensitive or more sensitive by such mechanical pressure. The stable materials used by Bridgman at some time previous to their detonation were in the condition of a sensitive high explosive due to the high pressure mechanically applied. And the cyanuric triazide which was already very sensitive, because of being in a state of strain, was made still

[40] *Proc. of the Am. Acad. of Arts and Sciences*, Vol. 71, No. 9, Jan. 1937.

more sensitive by an additional strain caused by mechanically applied pressure. It seems, then, that the strain theory of Von Baeyer has been strengthened and that the identity of a mechanical and a chemical force has been established in this case (see Note No. 8).

Bragg [41] says: "cleavage, hardness, electrical conductivity, thermal conductivity and refractive index are all directional properties of the crystal." The heat applied to a crystal will, therefore, be conducted through the crystal at different rates in the different directions of the axes. The strength of the crystal will also be different for the different axes. These seem good reasons to believe that suddenly applied heat or other types of energy will cause the crystal to break.

All of the data given here regarding explosives indicate that a temperature such as that produced by black powder is not very effective for the initiation of detonation. As has been stated in several of the references above slowly applied heat is not effective, whereas, suddenly applied heat is sometimes effective, and a sharp blow is effective when even suddenly applied heat is not. Also materials which are liquid do not detonate readily whereas the same or similar materials when in crystal form are very sensitive to detonation. What, then can a suddenly applied heat do that a slowly applied heat cannot do, and what can impact do that even a suddenly applied heat cannot do, and what could happen to a crystal which would not happen to the same material in the liquid form? It seems quite clear to the writer that the answer to these questions is that the former agency or condition in each case can cause breaking.

Now it has been shown that those materials which have rigidity and can be readily broken detonate readily and that materials which lack rigidity do not readily detonate. And that the type of energy which tends to break materials is the type of energy which causes detonation. Even the most insensitive explosives are detonated by the most effective breaking type of energy, i.e. sudden enormous pressure, such as is supplied by the explosion of a detonator (primary detonating explosive). It, therefore, seems the logical conclusion that detonation is brought about by the sudden application

[41] "The Crystalline State," Vol. 1, 1934, The Macmillan Co., N. Y.

of mechanical energy (detonation impulse), and is propagated by the regeneration of the impulse by the recombination of the broken material (products of detonation). In other words, the best initiator of detonation is, detonation itself.

It would seem to be worth while, at this point, to try to visualize, in more detail, the mechanism of the beginning of the impulse which is detonation. The only statements, known to the writer, regarding the detailed mechanism of adjustment that must take place at breaking are those of Bridgman [42] who says: "When rupture does occur, the parts of the body, and the atoms with them, receive accelerations, and there are unbalanced forces acting on the atoms. This may be stated in paradoxical fashion as follows: "A body does not break because there are unbalanced forces acting on the atoms, but there are unbalanced forces acting on the atoms because the body breaks."

We have presented evidence that the endothermic crystals are under a real mechanical strain, which is to say that they have potential energy due to that strain. When such a structure is broken there is a release of stress, in addition to the other unbalanced forces, acting on the atoms. It is probable that when a break occurs in an endothermic crystal, the chance that the atoms will find new positions of equilibrium without the complete destruction of the crystal are greatly restricted. I have found nothing in the literature relating to the nature of the surface of unsaturated crystals; however, according to Ward [39] their surface atoms must be more unsaturated (have higher energy) than those on the interior, for otherwise they would fly apart. It is probable that there is a delicate balance between such energies which changes as the size of the crystal changes and causes the great difference in sensitivity of the large and small crystals of these compounds.

Regarding the strained condition of the crystal, the words of Joffe [43] are significant. He says: "On removal of the stress, the main part of the strain disappears with the velocity of sound."

[42] *Jour. of Applied Physics*, Vol. 9, No. 8, Aug. 1938.
[43] "The Physics of Crystals," 1938, McGraw-Hill Book Co., N. Y.

When a explosive compound breaks, the release of energy will, then, tend to be at the velocity of sound, modified by the fact that the material, in the case of the explosive, is destroyed by the wave or impulse.

Now, then, the only requirement for the initiation of an impulse which will be propagated, is that the movement of the layer of atoms at the newly created surface have sufficient energy to reproduce the breaking impulse in layer after layer of atoms in the crystal.

Two instances in which the large crystals of endothermic compounds are more sensitive than the small crystals have been noted. This condition is probably true in general for the initiators, but has not been so forcibly brought to our attention for those compounds which have the quality in a less extreme degree. These two compounds, lead azide and cyanuric triazide, are very dangerous when the crystals exceed a certain size.

It is probable that a large crystal is more readily broken than a small crystal, and also probable that the breaking of a large crystal is more effective to start the breaking impulse. This last characteristic of the endothermic compound may give an insight of the detailed origin of the breaking impulse.

The breaking of a large crystal will produce a larger area of new surface than the breaking of a small crystal. It may be that a certain minimum area of new surface is required to allow a great enough disturbance of the atoms to start the impulse. For, as Bridgman says when rupture occurs, the atoms are subjected to unbalanced forces and move to new positions. In such movement the area of the new surface will probably be a critical factor for the amount and energy of the movement. We might visualize the condition in this way: The newly created surface will be bounded by a periphery where the new surface intersects the old surface. The change in the condition of the atoms at this periphery will be less than for the atoms in other parts of the new surface. The periphery of the new surface may act, then, as a node and the area of the new surface as a vibrating disk, the magnitude of which will to some extent control the amplitude of movement of the atoms. The critical amplitude will, then, be a movement which is great enough to break the weak, distorted bonds

of the multiple bond linkage. The continuance of the breaking will be assured by the release of energy by the recombination of the products of detonation (separate atoms) to molecular elements and compounds.

We have discussed the beginning of the disturbance which is detonation as a breaking in which the atoms receive accelerations and move to new positions. It can readily be seen that in such a disturbance of the atoms the amplitude of the movement must be less than the atomic distance and that the wave-length of the disturbance (impulse) must be not more than a few times the atomic distance. We know that the velocity of the impulse ranges from a low of about 2000 to a high of about 8000 meters per second. It can be seen, then, that the frequency of the disturbance must be in the range of the very high ultrasonic.

That such frequencies of the stress impulse may be very destructive can be shown by experiments which have been conducted by investigators in that field. Bergmann [44] speaking of the production of high frequency waves says: "The irregularity of vibration frequency leads to destruction of the plate (quartz crystal) for the elastic limit of the plate may be easily over-stepped in places of strong vibration, even when the load on the whole plate is relatively small." And continuing he says: "The powerful effect on the vessels wall also appears from the fact that some of the material of it is actually carried off by ultrasonic radiation in a finely divided form, and colloidally dispersed through the liquid." And later this statement is found: "The experiments above described relate to the dispersing powers of ultrasonic waves at the boundry surfaces between different substances, but this shattering power of the waves seems in no way dependent on such boundaries between phases, but also to work on a molecular scale. Szalay [45] succeeded in splitting up a number of highly polymerized molecules by ultrasonics of a frequency of 722 KHz. Starch was transformed into dextrine; gum arabic, gelatine, etc., were decomposed. The chain of a molecule of starch was split into five or six fragments. . . . Szalay further

[44] "Ultrasonics and their Scientific and Technical Applications," 1938, John Wiley and Sons, Inc., N. Y.

[45] *Z. Techn. Phys.*, **8**, 74 (1927).

succeeded in splitting up cane sugar into mono-saccharides, and also in breaking up paraformaldehyde and hexamethylene tetramine. . . . How this depolymerization by ultrasonics is brought about is still unexplained. The first necessity is to investigate the influence of the frequency of the sound."

Another observation which suggests a connection between detonation and ultrasonics is that of Finlay.[46] He says: "At the surface of the water after the explosion of a depth bomb a

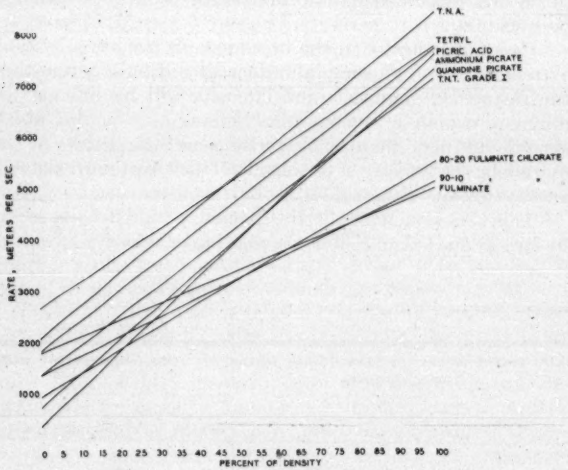


PLATE I

fine spray of water appears similar to the effect of supersonic vibrations when applied directly to water."

Then, if we assume that the impulse is of the order of frequency of the ultrasonic *how* high are those frequencies? It is possible, according to Bergmann, to generate frequencies as high as $5 \times 10^5$ KHZ. Slater [47] says that the maximum attainable frequency of the sound wave is $10^{13}$ cycles per second. He also says: "It can be shown theoretically that as

[46] *Scientific American*, March, 1940, Page 149.

[47] "Introduction to Chemical Physics," 1939, McGraw-Hill Book Co., N. Y.

this limit is approached, the velocity of the waves no longer is independent of the wave length. However, the change is not great; it changes by something not more than a factor of two."

This may well be an explanation of the high rates of detonation possessed by some explosives, which attain a velocity of more than 8000 meters per second, and may justify the belief that the frequencies of the detonation impulse are higher than any that have been generated experimentally.

Assuming, then, that the breakdown is produced by such extremely high frequency vibrations, it will be apparent that the frequency, amplitude and intensity will be different for different explosive compounds. Among the factors which should determine these characteristics are: the density of the materials, the velocity of the reaction following the breakdown and the atomic distances.

This variation in amplitude, frequency and intensity seems to furnish an explanation of the varying or selective effective-

TABLE I.

| Secondary Explosive. | Primary Explosive. Grams required for complete detonation. | | |
|---|---|---|---|
| | Cyanuric triazide. | Mercury fulminate. | Lead tri-nitro resorcinate. |
| Picric acid | 0.05 | 0.21 | 0.75 |
| Tetra-nitro-aniline | 0.09 | 0.20 | — |
| Tetryl | 0.04 | 0.20 | 0.80 × |

ness of initiating compounds. By selective effectiveness is meant the fact that one initiator may be very effective for one secondary explosive and much less effective for another, whereas another initiator may show the reverse condition. An example of such selective effectiveness is to be found in work reported by Taylor and Rinkenbach [48] the results of which are given in Table I.

[48] Reports of Invest. Nos. 2513, Aug. 1923, and 2533, Oct. 1923, Dept. of Int. Bur. of Mines.

It will be noted that the amount of mercury fulminate required for the initiation of tetryl, T.N.A. and picric acid are in the ratio of 1.00 to 1.00 to 1.05 while the quantities of cyanuric triazide are as 1 to 1.25 to 2.25. Lead tri-nitro-resorcinate is effective to initiate picric acid but does not initiate tetryl which requires the least quantity of the other initiators. Such action certainly cannot be explained by the total amount of energy released by the initiators. Rather, it seems obvious that the explanation must be found in some time-energy relation. In this connection, it is interesting to recall that the atomic distances of the multiple bond relation are less than for the single bond relation.

#### RECAPITULATION.

*The Multiple Bond.* For this type of bond the atomic distances are less than for other types of bonds. There is absence of free rotation about the double bond. The strength of the bonds is reduced by their deflection from their normal positions. There is chemical instability. The molecule is under a strain "comparable to the tension observed in the case of a bent metal spring." There is a *certain rigidity.*

*Rupture.* There is no general criterion of rupture. The phenomenon is specific with the material. The yield value of a crystalline body is rather sharp and by comparison the yield value of an amorphous body is unsharp. Many materials which are plastic under slowly applied loads are brittle when the load is applied suddenly. Rapid loading always favors rupture. There is little experimental evidence as to the effect that would be produced by the application of an impulse of the frequency of the ultrasonic, but it is probable that even very tough and plastic materials would be broken.

*Explosives.* Until the discovery of the unique ability of the first known primary detonating explosive (mercury fulminate) to produce a very intense impulse, no reliable agency was known for producing detonation of the high explosives. The action of this material is generally recognized as due to the production of very sudden and very high pressures.

The most sensitive explosives are seldom affected by slowly applied heat, whereas suddenly applied heat or a sharp blow cause detonation. Crystalline materials often detonate from

applications of energy which are not at all effective when applied to liquids or plastics.

Large crystals of some of the initiators are much more sensitive than the small crystals and sometimes detonate due to the application of mechanical pressure of a small magnitude.

The frequency of the impulse which produces and propagates detonation is undoubtedly in the upper range of the ultrasonic.

It is probable that the selective effectiveness of initiators is due to the difference in frequency, amplitude and intensity of the impulse produced by the different compounds.

We cannot always say, then, that one explosive is more sensitive than another, but that it is more or less sensitive to a certian impulse.

## CONCLUSIONS.

1. The initiators are endothermic crystalline compounds. These materials are inherently in a strained condition. They are chemically unstable. They are also more susceptible to breaking than other compounds. Their energy is released most reliably as detonation by the agency of a mechanical impact.

2. The instability due to the inherent strained condition can be made greater by the application of a mechanical stress from an outside source. This fact indicates that the inherent strain and the imposed strain are identical in their effect on stability.

3. Chemical reaction must be conceded to be, in most cases, a two-phase phenomenon, the first phase of which is breaking.

*Note No. 1.* The primary detonating compounds are those which detonate directly by the application of energy of almost any type, that is, by heat, friction or impact. They are so designated to distinguish them from the secondary detonating compounds which require a very special type of energy to cause them to detonate. In fact the only means which will reliably produce detonation of the secondary detonating compounds is the application to them of the detonation wave or impulse which has been set in motion by the primary detonating compounds. It is for this reason that the high explosives industry may be said to date from the time of Nobel's discovery of the effectiveness of mercury fulminate as a means of detonating the nitro glycerine explosives. Mercury fulminate is one of a very limited number of unsaturated endothermic crystalline compounds which are known as primary detonating explosives.

*Note No. 2.* "Why should a substance with the highest known melting point be of interest to the natural philosopher? First, because extremes are always interesting as testing generalizations drawn from limited ranges and extrapolated to conditions not yet tested."—McKenna, *Scientific Monthly*, June, 1938.

*Note No. 3. Resumé of the Breaking Theory.* Detonation is the progressive breaking of the structural bonds of a material by a stress which is transmitted as an elastic wave or impulse. The deformation which causes the breaking forces each portion of the material forward in the direction of the progress of the wave until the breakdown occurs. The breaking of the structural bonds destroys the elastic properties of the material so that there is no reciprocal movement as there is with an ordinary sound wave. Because of the lack of reciprocal movement the term impulse is preferred over wave.

The speed of the impulse is determined largely by the elastic qualities of the explosive compound.

The propagation of the impulse is maintained by the energy released by the reaction of the products of detonation which takes place in the region behind the impulse.

The sensitivity to detonation is largely controlled by the physical resistance of the explosive material to breaking and its ability to transmit a stress elastically.

The more insensitive the material, the greater the distortion and, therefore, the greater the forward movement of the material before failure of the structure occurs (see Note No. 6). The greater the forward movement before failure the greater will be the amplitude of the impulse. Therefore, the more insensitive explosives will have the larger increment of increase of the rate of detonation with increase of the density (apparent density) because of the greater forward movement of the explosive before breakdown.

This condition is shown in Plate I in which the rates of detonation are plotted against percentage of the specific gravity of the materials and extrapolated to zero. The formula for these curves may be written as $Y = (a \times a')X \times b$. The equation represents the sensitiveness value of the explosives. It will be noted that the more insensitive explosives have the steeper curves.

*Note No. 4.* In any chemical reaction, it must be conceded that instability must precede the reaction, for the existing chemical bonds, in most cases, must be broken before new bonds can be established. Many authorities believe that intermediate unstable compounds are formed during catalytic reactions and others that the adsorption of catalytically active surfaces cause instability by deflection of the valence bonds from their normal position. Such deflection of the valence bonds is an inherent property of the multiple bond relation. The importance of such reasoning can be understood when it is recalled that W. Ostwald contended that there is no kind of chemical reaction which cannot be influenced catalytically. In gas reactions the collisions of the molecules is considered to be an important factor controlling the rate of reaction. Such collisions would act as any other distorting force to displace the bonds from their normal positions, again producing instability by the same means. However, whatever the *modus operandi* of the reaction, instability must precede the reaction and instability (chemical and/or physical) is the dominant quality of the known compounds which have multiple bonds. It is, therefore, the opinion of the writer that those who have been studying such phenomena as crystal structure, catalysis,

active and inactive molecules and chemical reactivity have missed one of the most fruitful sources for understanding of these subjects by their neglect to study the highly sensitive explosive compounds, which are extreme cases of chemical and physical instability. For as stated in Note No. 2 extremes delineate and establish the condition of the intermediate.

*Note No. 5.* This reference seems to me to represent a ready made description of detonation, less the source of the suddenly applied very high load. The separate atoms are, then, the products of detonation. When these separate atoms recombine to form molecular elements or molecular compounds they release the energy of recombination and produce the very high load which is suddenly applied to continue the cycle of breaking and complete the picture of detonation. However, it is believed that in detonation the breakdown is not always to the condition of separate atoms. This would probably occur only in perfect detonation, such as that of the extremely sensitive materials. It is probable that in some cases the weaker bonds only are broken and that what is known as low order detonation is a case of such partial breaking. Low order detonations occur with the military high explosives, some of which are very insensitive materials, when for some reason the booster delivers a shock not quite sufficient to cause complete detonation. In low order detonation it is probable that enough of the bonds are broken to start a very rapid chemical reaction, which may consume the entire charge or leave a portion entirely unchanged. The final reaction products, however, are entirely different from those produced by high order detonation. This shows that partial or incomplete breaking of the bonds is just as much a first phase of chemical reaction as is the complete breaking. A study of the extreme case here again gives a better picture of the intermediate.

*Note No. 5A.* The first three of these statements are so fundamental that they are instinctively apparent to everyone and so may be called "truisms." Applying to the first is a line from "The Broken Man" by Elinor Wylie: "And you were broken but you were not bended." The second is supported by the Chinese story of the old man who had lost all of his teeth but who still had all of his tongue. The third by the safety factors used by engineers for materials of construction. The factor for suddenly applied loads is many times as high as for a steady load and higher still for brittle materials like cast iron as compared to steel.

*Note No. 6.* A consideration of this fact prompts a revision of the breaking theory. It will readily be seen that a very violent reaction of the products of detonation behind the breaking impulse will cause breaking of the explosive material with a lesser deformation before breaking than a less violent reaction. Therefore, the increment of forward movement of the explosive will be less, and the curve representing the rate of detonation plotted against density will be flatter for those explosives whose products of detonation react more violently. Thus the *sensitiveness value* will reflect the sensitiveness which is due to the high reactivity of the products of detonation as well as that due to the physical properties of the explosive.

*Note No. 7.* A rubber policeman is a glass rod with a rubber tip fitting snugly over its end, which is used by chemists to remove precipitates from chemical apparatus.

*Note No. 8.* This offers an opportunity to define more clearly the difference between the *primary* and *secondary* detonating explosives. The primary detonating explosives are in an inherently strained condition "like a bent metal spring" or a gun that is cocked, while the secondary detonating compounds must have a specialized force from an outside source act upon them to bring them into the strained condition. The primary detonating explosive or initiator is the only reliable agency for supplying such a specialized force.

*Note No. 9.* On the contrary small crystals of exothermic explosives are more sensitive to detonation than the large crystals of the same compounds. However, the difference in sensitivity in their case is of less consequence and not so great as is the case with the endothermic compounds. The slightly greater sensitivity of small crystals of exothermic compounds probably can be accounted for by their greater energy content derived from a greater area of unsaturated surface.

The exothermic crystals also break when suddenly subjected to a stress. However, due to their inherent stability, detonation does not result unless the stress is of the proper amplitude and frequency such as is supplied by an initiator. Otherwise, the breaking merely produces a new surface which acquires energy by becoming unsaturated.

**New Oil Aids Production of Low-Alloy Steels.**—Gulf Oil Corporation technologists have developed a quenching oil which aids war production of low-alloy steels and which is helping to offset the shortage of alloys, most of which are imported. As applied to many types and shapes of steels, it has shown spectacular effects. With a cooling speed more nearly approaching that of water through the higher temperature range, this product improved physical properties in most of the steels that were tested. Yet it retained the minimum tendency toward distortion and cracking that is characteristic of all quenching oils. To check and double check the new product, test after test was carried out on equipment especially developed for the purpose at the Gulf Research and Development Company's Laboratory near Pittsburgh. This utilizes a highly sensitive oscillograph to measure the change in the electromotive force of a thermocouple in the steel as it is being quenched. A photographic record of the movement of the oscillograph is made on a sensitized tape, which is driven by a motor through a camera at a rate of six inches per second. From this tape the temperature change in the steel during any interval of one two-hundredth of a second can be determined. Gulf's researchers found that where ordinary quenching oils will always achieve the desired hardness in heat treating a steel of a given specification and where water was being used without distortion or cracking there appears to be no reason for using this new super-quench oil. But between these two extremes, there were hundreds of applications involving all kinds of alloy and plain carbon steels where the new product would prove superior to any other known quenching medium. According to a Gulf technologist, the new oil promises to save thousands of tons of critically scarce alloys that go into certain kinds of steel now in abnormally great demand for war material. In the past it has been necessary to increase the alloy content of steel to get the desired physical properties because of the limitations of the quenching oils. Now these alloys are scarce. By using the new product metallurgists have found it possible, in many cases, to get the same hardness and toughness in steels containing lesser amounts of vital alloying elements.

R. H. O.

# YOUR PLACE IN THE WORLD OF TOMORROW.

BY

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(The following is a series of three talks given for young people in the Christmas Week Lectures of THE FRANKLIN INSTITUTE, December, 1942.)

Our discussion of your place in the world of tomorrow will fall into three parts, each part with a title somewhat arbitrarily chosen. First we shall talk about intelligence, what it is, who has it, and what can be done about it. Secondly, we shall consider testing for talent, a consideration of the special aptitudes and abilities possessed by individuals. And finally, we shall relate these ideas and some others to the problems that face you in choosing a career, a career which will fit into the world of tomorrow. Perhaps the title of all of these talks should really have been *Choosing a Career*, because each idea that we are to deal with is intended to contribute something to the choosing of your career, and it is only logical that this career should be one appropriate to the world of tomorrow.

## INTELLIGENCE.

To those interested in a career, there is one significant fact which should be very comforting, that is that tomorrow's world will be your world—will be a world created by and for people; man has more and more through the centuries turned the vast riches of the earth to his use and dominated its powers in his own interest. More than that, we hope that tomorrow's world will be a world for our kind of people, and not for the few only, but for the majority of the people. By and large, the inhabitants of this world have not changed much since the dawn of recorded history and perhaps before. They have not changed materially during my lifetime and you can rest assured that they will not change greatly during the course of your life or the life of your children. Each generation ex-

presses itself in a slightly different way and the preceding generation has always made a note of this and always expressed great concern for the future of mankind, but actually,

FIG. I.

change comes very slowly and the change in man himself so slowly as to be almost imperceptible from century to century.

If we are to predict the future accurately and find our place with some confidence, it is best that we examine the breadth

of human potentialities for change, and also human limitations. It is fortunate from my point of view that we are having our meetings in The Franklin Institute, for it illustrates one aspect of this problem very well, that is, the breadth of human potentialities for change. I engaged the services of a very clever artist to illustrate the other aspect, namely, the limitation of human variability.

First, let us consider Mr. A (Fig. I), a gentleman from many generations back, far beyond the dawn of history. As you see, he or his forebears have already solved the problem of increasing the effectiveness of his blow as evidenced by the stone hammer on the ground beside him, and now he is producing a fire. This fire will warm his cave, furnish light by which he may make more weapons, improve the taste of his food, furnish power for disintegrating substances, etc. The artist has depicted Mr. A in the process of testing an idea, or the relationship between two ideas. First, he has observed that his hands are warmed while polishing the handle of his hammer; also, that a similar sensation is experienced when he places his hand over the open fire. Mr. A decides to try to produce fire by friction. He selects his materials, sets the stage and, perhaps after many failures, he succeeds. He now has the principle of artificially produced fire. Others imitate, but few understand the principle. On the part of other members of his society, failure to produce the phenomenon will be attributed perhaps to evil spirits, and success to the gods or some favorable sign. But nine times out of ten Mr. A will succeed, and his success is due to some trait in Mr. A's nature which sets him apart from all those with whom he is associated. Mr. A is an extremely intelligent person.

Mr. B (Fig. II), on the other hand, is using one of the perhaps two or three basic principles inherent in all machines. This is called the machine age; if so, it is only because we are devising new applications for these old principles. Archimedes put this particular principle in words and invented a slogan to sell it. It was he who said, "If you give me a lever long enough and something to rest it upon, I will move the world." He may very well have been the first person really to understand the principle underlying the behavior of Mr. B. Mr. B is a powerful man but, powerful though he is, he would be

unable to move this heavy object without the aid of a simple machine, in this case a lever. Mr. B, or whoever discovered this principle and understood its full import, had a power over nature which was greater than that of the fabled giants. He was a very intelligent person.

FIG. II.

Mr. C (Fig. III) is demonstrating still another principle of mechanics, perhaps the only other one—the inclined plane. He has also discovered how to reduce the friction which I mentioned in the problem of Mr. A by placing rollers under the heavy block. However, unless the inclination of this plane is very slight, Mr. C could not move this massive stone by himself. Therefore he discovers another aid to the solution of this problem, the getting of others to help him. Mr. C has not only a principle of mechanics but also the principle which underlies all social organization. Mr. C is a very intelligent man.

In the center of the large chart (Fig. IV) we see Mr. D. Mr. D is very happy, as you can see by the tilt of his head

and his jaunty stance. He has just received a bonus. He is an estimator, he works in an office with many other men. He is married. He lives in a small house in the suburbs. He has

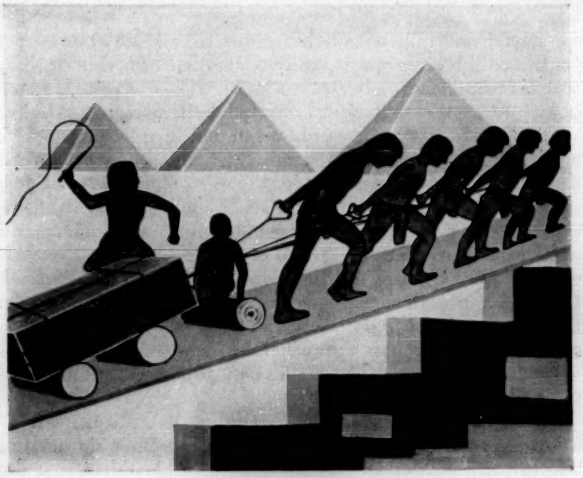

FIG. III.

FIG. IV.

two children and a mortgage. He uses language and numbers and a slide rule, and spends all day putting little marks on paper. He did not invent any of the systems that he uses, but he learned to use them very well. He may even under-

stand a few of the principles on which these systems are based, but he need not. He is just an average man, like the sixty other little men here in the center. Why sixty other little men? Simply because if we should stand at a busy intersection and gather in the first hundred men that passed by, we should find that out of that group about sixty, or 60 per cent., would behave in a manner very similar to Mr. D. None would be outstanding because of an excess of ability, or lack ability to such extent that he would be considered inferior; that is to say, sixty men out of a hundred are so nearly alike and their similarities so outweigh their differences that they can be considered as a group.

However, Mr. E, who stands at the extreme right of our chart, the very superior looking individual who looks down upon Mr. D, represents a group of twenty out of a hundred whose differences are much greater than their similarities, these differences demonstrated in economic superiority. Mr. E's real superiority is probably due to the fact that he understands, as well as uses the principles, which are basic to his field of endeavor so well that he can adjust his methods to meet changing conditions.

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Mr. F, at the other extreme, the man down to the left, is also one of twenty whose differences exceed their similarities but in the direction of inability, and these differences are demonstrated in economic inferiority. Mr. F and his group find it increasingly difficult to cope with such complex situations as principles and laws. It is in some cases possible to train Mr. F to use a match, to make a fire, to use a lever and lead a band, to work safely and, in some cases, efficiently, but he will never understand the principles of combustion, nor the lever, nor the basis of social organization.

We have been talking about men who differ principally in a single human trait; this trait is sometimes labeled as intelligence. By definition, it is the ability to solve problems successfully. There are many different problems and there are many different types of intelligent behavior. Therefore a man is intelligent when he displays intelligent behavior in solving a problem in some particular field. Many of us are not capable of solving problems in several fields but are restricted to the solution of problems which involve real objects, and this

is sometimes called mechanical intelligence. If the problem involves symbols rather than real objects, it is usually called abstract intelligence. If the problem involves the consideration of other human beings in order to insure its success, the solution of this problem requires what is generally called social intelligence.

There is very little relationship between these three broad fields in which intelligent behavior can be displayed. Some individuals do well in all three, others in one or more.

We hear a great deal today about the Intelligence Quotient or I.Q. which is only in the narrowest sense of that word a measure of intelligent behavior, and then in the single field of abstract intelligence. It seems to be a very useful instrument because it lends itself to apparent quantitative determinations expressed in the number series from zero to possibly two hundred. The evaluation of the Intelligence Quotient is determined by a somewhat devious route. It usually consists of a series of questions administered verbally to a random sampling of children of various ages, and by this process the average response for each age is determined. Instead of crediting the child with a correct response, he or she is credited in months, the total credit establishing the Mental Age. If the Mental Age coincides with the chronological age, the two may be expressed as a ratio and for convenience, this ratio is multiplied by one hundred. So that the Intelligence Quotient, or I.Q., is expressed

$$\text{I.Q.} = \frac{\text{Mental Age}}{\text{Chronological Age}} \times 100.$$

For example, a child 120 months old who receives a credit of 120 months on the test will have an I.Q. of 100, indicating that in this respect the child is normal for his age. If, however, the child is 120 months old but has only 60 months credit, the I.Q. is 50. Likewise, if the child is 60 months old and gets a credit for 120 months, he is accelerated in this respect and would receive an I.Q. of 200.

Dr. Lewis E. Terman has estimated from such evidence as the following letter that Sir Francis Galton probably had an Intelligence Quotient of about 200. You can see from the nature of the behavior as displayed by this letter that

Francis Galton at the age of five years handled symbolic material in general about as well as the average ten year old boy.

My dear Adele:

I am 4 years old and I can read any English book. I can say all the Latin Substantives and Adjectives and active verbs besides 52 lines of Latin poetry. I can cast up any sum in addition and can multiply by 2, 3, 4, 5, 6, 7, 8, (9), 10, (11). I can also say the pence table. I read French a little and I know the clock.

Francis Galton
February 15, 1827

On the other hand, Dr. Terman estimated the I.Q. of Michael Faraday as 105. However the life of Michael Faraday indicates that he was the discoverer of electro-magnetic induction, of the process of alloying steel, he was the first to liquefy chlorine, and he led in many other notable scientific achievements. However, he had some difficulty with symbolic material, he spent the greater part of his youth in a desperate attempt for self-improvement; he was never a great writer, and was said to have been always a poor mathematician.

The difference in the value of these two numbers with respect to these two men—that is, 105 and 200—does not represent relative differences in intelligence but relative differences in their ability to solve problems using the same kind of material. In general, both were men of the first water, both were in a class with Mr. A, both were very intelligent.

At this point, then, we may say that intelligence is a way of behaving and that we all have this trait to some extent. In our second discussion I hope to make it clear to you how individuals differ in not only this trait but in all other human traits of this nature. We still have not touched upon the question of what can be done about it, that is, what can be done about it in case we are lacking in intelligence. At this point, I would simply answer that question by saying that education is the only solution. It took primitive man thousands of years, perhaps, to discover the solution to the problem of artificially produced fire. Today the average or less intelligent members of our culture can be trained in a few minutes to produce the same results. This means that few may under-

stand the intricate principles upon which new developments of science depend, few names are on the honor roll of great discoverers and creators; yet most of us, with adequate training can use the products of these great minds, whether those products be material things or ideas.

### TESTING FOR TALENT.

Today we are going to consider the problem of testing for talent and it would, perhaps, be well for us to indicate the precise meaning of the word "talent." In general it applies to a superior aptitude, and an aptitude is one or more traits of character which are measurable and which are regarded as an indication of an individual's ability to acquire with training some specific knowledge or skill. Testing for talent therefore resolves itself into the problem of measuring groups of individuals in order to discover the range within the group of certain important traits, and finally to establish the position of an individual within this group. The greater the relative superiority, the greater the talent possessed.

Lincoln is supposed to have said, "God must have loved the common people because he made so many of them." If he meant by "common" the average person, he was expressing a very fundamental concept in applied psychology and one which is generally not too well understood. For if it is possible to measure a human trait—and it is possible to measure many, both physical and mental—the average is the theoretical dividing line which separates the upper half of the group from the lower. This really leaves no room for the average person but separates all into two groups, one group superior to 50 per cent. of the population, and the other inferior to 50 per cent. In practice, however, we observe that only a few in the upper 50 per cent. seem to possess the trait in such quantities as to be considered outstanding. Likewise, only a few in the lower group lack the trait to the extent that they are handicapped by the lack of it.

Let us consider the trait of height. How tall must a man be before he is considered tall or outstanding, and how short before he is considered to be outstanding? That is to say, on what height dimension do you acquire the nickname of Shorty or of Slim? The height of the average adult male in the Brit-

ish Isles, which is a rather homogeneous population, is about 67 inches, and approximately 60 per cent. of the population will not vary sufficiently to be considered tall or short. If, however, we mark off the point where tallness begins to make itself evident, we will find that those individuals are 70 or more inches tall. And, conversely, where the lack of this

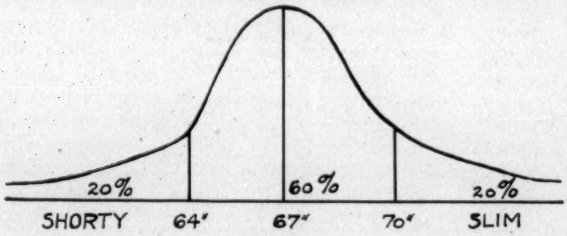


FIG. V.

trait begins to make itself evident, the individuals will be 64 inches or less. Referring to our original definition of talent, we might say that in this trait, the talented are those who have 70 or more inches of height. This, however, depends entirely upon our orientation. If we are, for instance, considering this trait with respect to a center for a basketball team, we would consider only individuals in the upper twenty per cent. as superior for this particular activity. On the other hand, if we are considering applicants for coxswain for a varsity eight, only those in the lower twenty per cent. would be considered superior on this particular trait, because it is the absence of height in which we are interested. This means that the test we use as a measure must be very carefully related to and considered in the light of the goal for which we are measuring. The very trait within the individual which spells success in one field might mean failure in another field; and conversely, the traits which compel us to give up hope of competing in one line of endeavor may, when thoughtfully considered, point toward great likelihood of success in another line of endeavor.

Success in scholastic competition depends to some extent upon certain mental traits which are measurable and therefore highly predictable of success in this activity. Tests of these

traits are usually called tests of scholastic aptitude. Again when we consider the results of such tests—as in every case where we are dealing with growth resulting from the impact of the environment upon the individual—we have this same normal distribution; sixty per cent. of the general population are so nearly alike in the traits that they can be considered within the normal group; twenty per cent. possess the necessary traits in such quantity that unusual success is predicted in scholastic competition; another twenty per cent. are so lacking in these qualities that they are more or less excluded from competition in this particular activity.

In addition to the more familiar tests in the scholastic field, there is a great group of predictors in other fields, some rather general, some highly specific. Tests of mechanical aptitude, social aptitudes and motor skills are all considered indicators of the individual's success in development along these particular lines. Moreover, at the present time, there are many measures for aptitudes such as teaching, law, engineering, languages, general science, mathematics, chemistry, music, art, and commercial pursuits, etc.

### CHOOSING A CAREER.

A career is the portion of one's life which is directed toward furthering success in a chosen vocational or professional field. In choosing a career, in general we must think about three aspects of the problem: first, the world of work, that is, what careers or fields of activity are there which we can enter; second, our personal qualifications; and third, the procedure by which we prepare ourselves to enter into the field of activity which we have chosen.

Let us consider the first. How does one get information concerning the world of work? First, by observing men and women at work. This we begin to do casually in our earliest years, and in some cases the interests which later form the basis of the choice of an individual's career are built upon such early observations. As we grow older and begin to think of ourselves in relation to jobs, we observe people at work more closely, analyzing the things they do, watching their apparent interest in their jobs and trying to imagine our own interest in those jobs, measuring ourselves against them. Secondly, we get information about the world of work by reading.

Again, some of this information may be acquired rather casually, as we chance to read of the lives and work of men who have been outstanding in their fields. But when we come to the point of making a definite choice of career for ourselves, it is well also to consult some of the standard books describing fields of work for men and women, books which present in well-organized fashion a number of possible choices, with some discussion of requirements for success, and a brief presentation of the probable advantages and disadvantages of each. And finally, we get information concerning the world of work by consulting experts in the various fields of endeavor in order that we may get first-hand information as to the personal qualifications and exact preparation that is necessary in their opinion for success.

Next the personal factors must be considered. It is well at the outset to subject ourselves to all types of aptitude tests in order that we may uncover any specific abilities or defects that could help or hinder us in our chosen profession, or which might lead us to choose one profession or steer us away from another. Naturally, a wise use of these aptitude test results is possible only if the individual has a very thorough understanding of the abilities required for success in different careers. It is at this point that our study of the world of work, already mentioned as the first step, becomes most valuable to us, and it is at this point that we are most apt to need the help of experts. It is well to consider various temperamental traits which are expressed in different ways by different individuals. It is well to know in advance whether we tend to be dominant or submissive in face to face situations with our fellow workers. Our emotional pattern, the degree of submissiveness, our self-confidence, and many other personal traits can be examined by those who are competent in the field of vocational guidance, and the results evaluated in terms of our choice of a career. Our interest pattern might well be considered, also, in order that we may determine the relationship between our general interests and those of persons already considered as successful in our chosen field. In general our interest patterns cluster around a nucleus of activities that might be considered as academic, commercial, scientific, social, mechanical, and artistic.

It must be remembered, however, that within any vocational or professional group many interest patterns may be found. For instance, in engineering the individual with academic interests may teach; the individual with social interests may find a career in personnel; the individual with purely scientific interests may very well devote himself to research in engineering; and the person with a liking for the commercial may use his engineering training in the manufacturing, selling, or sales promotion fields.

Although aptitude tests may be used to predict success in training, success itself is due to more than just aptitude. Therefore, when we are confronted with the choice of a career it is well for us to examine every shred of school and work history available in order to throw light upon our ability to accept training as far as we have already endeavored to enter into these competitive fields. For instance, if you are considering a professional field which will require college training, some indication of your ability to accept college training may be gathered from a review of your high school record. Many institutions of higher learning, for example, will not admit students from high school unless they are in at least the upper 50 per cent. of their graduating class. Others restrict their entrance to the upper third, and still others to the upper 25 per cent.

One should consider well the health and physical requirements for his chosen profession. For example, the handicap of poor vision eliminates many well-trained individuals from long time participation in certain fields, or a mild cardiac condition may prevent a girl's entering physical education or nursing, a profession which is quite strenuous, but not be a serious handicap in the field of language teaching.

Certain social, political, and labor groups exercise restrictive measures which will sometimes prevent otherwise well qualified individuals from participating in an activity because of their inability to break into these above-mentioned groups. It is well for all of us to make sure before we choose our life work that there are no insurmountable barriers of this nature placed before us.

The economic status of the individual and the family must also be given consideration when selecting or advising in the

choice of a career. This situation is very complex and leads to many paradoxical situations. For example, the boy with a talent in the educational field who is forced by financial limitations to work his way through college may, if he has strength and luck, arrive, as did a friend of mine, at the end of his senior year with $2.35 to the good after supporting himself throughout the four years of college and paying his tuition through his own efforts. A person with mediocre ability but considerable financial backing and normal health may succeed in a training situation and in the professional activity that follows whereas the same individual with poor health or no financial backing would be almost sure to fail despite somewhat greater ability.

We have a right to expect, in fact, to demand, a certain satisfaction from any job or profession. That satisfaction may have its roots in the yearly earnings that we may expect, or in the service we render, or in the material we create. Therefore it would be well for us to consider every career in the light of the financial reward, the amount of service we render and whether or not that service will supply us with increased job satisfaction; or possibly whether the materials we create (even though their commercial value may be such that our earnings are reduced) will compensate in job satisfaction the loss of the financial reward.

Once you choose a profession, you identify yourself with a group, you compare yourself with that group and are compared with that group, and no other group matters to you. To a member of a third group, it may appear that plumbers are in general on a lower vocational level than, say, members of the medical profession; however, to be the best plumber in the community will bring a great deal more satisfaction and probably a great deal more financial reward than to be the poorest physician in the community. In the first instance the man is the tops in his particular group, and it is with that group that he is competing; whereas in the second instance, the man has failed in competition and is consequently looked down upon by all his associates. In order then that we may wisely choose, it is well for us to consider all of the factors mentioned and endeavor to chart the course which would most nearly assure us of at least average success in our chosen field.

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# PERIODIC PROCESSES IN FREE PSEUDO-LINEAR OSCILLATORY SYSTEMS.

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1. *Exposition of the Problem.* The present paper deals with a mechanical or an electrical system with many degrees of freedom governed by pseudo-linear differential equations. On the basis of the theory due to H. Poincaré, periodic solutions representing an important type of steady oscillations are obtained. These solutions are then applied to explain the self-excited oscillations of the follow-up systems.

Let

$$\sum_{k=1}^{n} f_{jk}(D, \mu)x_k = \mu\varphi_j(x_1, \cdots, x_n, \mu), j = 1, \cdots, n \qquad (1)$$

be the differential equations of the system. The letters $x_k$ denote the unknown coordinates of the system depending on the time variable $t$,

$$D = \frac{d}{dt} \qquad (2)$$

is the differential operator, $\mu$—a small parameter; $f_{jk}(D, \mu)$ and $\varphi_j(x_1, \cdots, x_n, \mu)$ are given continuous functions of the variables involved, $f_{jk}(D, \mu)$ being with respect to $D$ simply polynomials of finite orders, usually quadratic. Since the non-linear terms in the right-hand sides appear with the factor $\mu$ the considered system is referred to as pseudo-linear.

The periodic solutions for the special case when only one of the functions $\varphi_j$ is other than zero are due in essence to H. Barkhausen [1] and G. Möller [2] and were developed by different

[1] H. Barkhausen, Elektronenröhren, 2. Bd., 2. Aufl., Leipzig, 1926.

[2] G. Möller, Die Elektronenröhren und ihre technischen Anwendungen, Braunschweig, 1920.

methods by N. Kryloff and M. Bogoliùboff [3] as well as by A. Andronow and S. Chaikin.[4]

2. *The Simplified Linear System.* We shall suppose that for all values of $\mu$ under consideration the system (1) can be solved to yield explicit expressions for the highest derivatives $D^{n_k}x_k$ of the unknowns $x_k$. In order that the simplified linear system

$$\sum_{k=1}^{n} f_{jk}(D, 0)x_k = 0, \qquad j = 1, \cdots, n, \tag{3}$$

corresponding to $\mu = 0$ admits a periodic solution $x_k = C_k e^{i(\omega t+\epsilon)}$ as well as $x_k = \bar{C}_k e^{-i(\omega t+\epsilon)}$ we shall assume that for this value of $\mu$ the determinant $\Delta(D, \mu)$ of the operational matrix $f(D, \mu) \equiv \|f_{jk}(D, \mu)\|$ can be expressed in such a form as

$$\Delta(D, 0) = (D^2 + \omega^2)\Delta_1(D, 0). \tag{4}$$

We shall restrict our study to the ordinary case when $\pm i\omega$ are simple roots of the polynomial $\Delta(D, 0)$ and when no integer multiplies $L\omega$ coincide with any of the fundamental frequencies of the system (3); i.e.,

$$\Delta_1(\pm iL\omega, 0) \neq 0, \qquad L = 0, 1, 2, \cdots. \tag{5}$$

For the applications, in particular to regulation problems, it is of importance, that no further assumptions relative to the nature of the roots of $\Delta(D, 0)$ other than $\pm i\omega$ are made; for example, they may possess considerable real parts, corresponding to the strongly damped partial oscillations.

If $F_{jk}(D, \mu)$ denotes the co-factor of $f_{jk}(D, \mu)$ in $\Delta(D, \mu)$, one at least of the quantities $F_{jk}(\pm i\omega, 0)$ must be different from zero. Let

$$F_{lm}(\pm i\omega, 0) \neq 0, \tag{6}$$

so that we can assume

$$C_k = s\frac{F_{lk}(i\omega, 0)}{F_{lm}(i\omega, 0)}, \qquad \bar{C}_k = s\frac{F_{lk}(-i\omega, 0)}{F_{lm}(-i\omega, 0)}, \tag{7}$$

[3] N. Kryloff and M. Bogoliùboff, New methods of Non-linear Mechanics in Their Application to the Study of the Functioning of Electron Tube Generators, Moscow, 1934 (Russian).

[4] A. Andronow and S. Chaikin, Theory of Vibrations, Moscow, 1937 (Russian).

where $s$ denotes a certain quite determinate factor of proportionality; the bars over the letters are employed as usual to denote the conjugate complex quantities. We have obviously

$$C_m = s. \tag{8}$$

The set of real functions

$$x_k^*(t) = \tfrac{1}{2}A[C_k e^{i(\omega t+\epsilon)} + \bar{C}_k e^{-i(\omega t+\epsilon)}] = ARe C_k e^{i(\omega t+\epsilon)}, \qquad k = 1, \cdots, n, \tag{9}$$

containing two arbitrary constants $A$ and $\epsilon$ can also be employed as a periodic solution of the simplified linear system. With

$$C_k = L_k + iU_k \tag{10}$$

this can be expressed alternatively as

$$x_k^*(t) = A[L_k \cos(\omega t+\epsilon) - U_k \sin(\omega t+\epsilon)], \; k = 1, \cdots, n. \tag{11}$$

*3. Process of Construction of the Periodic Solution for* $\mu \neq 0$. Let a new independent variable

$$\tau = (1 + \alpha)t \tag{12}$$

be introduced, where $\alpha$ must vanish with $\mu$. The complete determination of this constant in terms of $\mu$ is to be found later. It follows

$$D = (1 + \alpha)D', \tag{13}$$

where

$$D' = \frac{d}{d\tau}. \tag{14}$$

If a periodic solution of the frequency $\omega$ in $\tau$ be obtained, its frequency in $t$ would be

$$\omega_1 = (1 + \alpha)\omega;$$

the quantity

$$\alpha = \frac{\omega_1 - \omega}{\omega}$$

expresses then the frequency correction which must be introduced according to H. Poincaré,[5] in all cases when the time variable does not enter explicitly in the differential equations.

[5] H. Poincaré, Les méthodes nouvelles de la mécanique céleste, T. I, p. 89, Paris, 1892.

In conformance with (13) the simplified linear system corresponding to $\mu = 0$ must now be written formally as

$$\sum_{k=1}^{n} f_{jk}(D', 0)x_k = 0, \qquad j = 1, \cdots, n \tag{15}$$

and its periodic solution as $x_k^*(\tau)$, $k = 1, \cdots, n$. For $\mu \neq 0$ all differences $x_k - x_k^*$ formed by means of the general integral as well as their derivatives up to and including $D'^{(m_k-1)}(x_k - x_k^*)$ may be considered as functions of $\tau$, of their own initial values $a_1, \cdots, a_N$, of $a_{N+1} = \alpha$ and of $\mu$:

$$\begin{aligned} x_1 - x_1^* &= \xi_1(\tau, a_1, \cdots, a_N, a_{N+1}, \mu), \\ D'(x_1 - x_1^*) &= \xi_2(\tau, a_1, \cdots, a_N, a_{N+1}, \mu), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ D'^{(m_n-1)}(x_n - x_n^*) &= \xi_N(\tau, a_1, \cdots, a_N, a_{N+1}, \mu). \end{aligned} \tag{16}$$

We have $\xi_i(0, a_1, \cdots, a_{N+1}, \mu) = a_i$ and the conditions for periodic solutions are

$$\xi_i\left(\frac{2\pi}{\omega}, a_1, \cdots, a_{N+1}, \mu\right) - a_i = 0, \qquad i = 1, \cdots, N. \tag{17}$$

All $\xi_i$ vanish when $a_1 = \cdots = a_{N+1} = \mu = 0$ and the conditions are satisfied. For an assigned value of $\mu$ different from zero they must be solved with respect to $a$; one initial value $a'$ out of $a_1, \cdots, a_N$ remains indeterminate and plays the part of a phase constant. We may take it to be zero since $x_k^*(\tau)$ contains already the constant $\epsilon$ of this kind.

The actual construction of the periodic solution can be effected step by step and the process will now be explained on the example of the first approximation. Suppose that $f_{jk}(D, \mu)$ and $\varphi_j(x_1, \cdots, x_n, \mu)$ admit continuous derivatives with respect to $x_1, \cdots, x_n, \mu$ and that (17) can be solved. Then a periodic solution of (1) near to $x_k^*(\tau)$ exists and is for its part a continuous function of $\mu$ admitting continuous derivatives of the corresponding orders with respect to this parameter. This follows from the general theorems on the dependence of the integrals on the initial values and parameters and on implicit functions. We may therefore differentiate (1) with respect to $\mu$ with consideration of (13).

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Putting

$$y_k = \left(\frac{\partial x_k}{\partial \mu}\right)_{\mu=0}, \quad k = 1, \cdots, n, \qquad C = \left(\frac{d\alpha}{d\mu}\right)_{\mu=0}, \tag{18}$$

where $y_k$ are also periodic functions, we obtain for $\mu = 0$

$$\sum_{k=1}^{n} f_{jk}(D', 0)y_k + \sum_{k=1}^{n}\left[\frac{\partial f_{jk}(D, \mu)}{\partial D}\right]_{\substack{D=D' \\ \mu=0}} D'Cx_k^*$$

$$+\sum_{k=1}^{n}\left[\frac{\partial f_{jk}(D, \mu)}{\partial \mu}\right]_{\substack{D=D' \\ \mu=0}} x_k^* = \varphi_j(x_1^*, \cdots, x_n^*, 0), \; j = 1, \cdots, n \tag{19}$$

or

$$\sum_{k=1}^{n} f_{jk}(D', 0)y_k = -\sum_{k=1}^{n} \rho_{jk}(D', 0)x_k^* + \varphi_j(x_1^*, \cdots, x_n^*, 0), \qquad j = 1, \cdots, n \tag{20}$$

with

$$\rho_{jk}(D, \mu) = \frac{\partial f_{jk}(D, \mu)}{\partial D} D \frac{d\alpha(\mu)}{d\mu} + \frac{\partial f_{jk}(D, \mu)}{\partial \mu}. \tag{21}$$

*4. Periodicity Conditions for the First Approximation.* Retaining still the hypothesis of the existence of the periodic solution we proceed to the determination of $y_k$ from (20) as periodic functions of $\tau$ of the frequency $\omega$. The inverse problem of proving the existence of a formally computed periodic solution will be solved in the sequel (par. 6) by itself in building the higher approximations,—at least for the case when $f_{jk}(D, \mu)$ and $\varphi_j(x_1, \cdots, x_n, \mu)$ are analytic.

To the stated purpose we note that for any polynomial $P(D)$ we have

$$P(D)e^{i(\omega t+\epsilon)} = P(i\omega)e^{i(\omega t+\epsilon)}$$

and therefore by (9)

$$\rho_{jk}(D', 0)x_k^*(\tau) = \tfrac{1}{2}A[\rho_{jk}(i\omega, 0)C_k e^{i(\omega\tau+\epsilon)} + \rho_{jk}(-i\omega, 0)\bar{C}_k e^{-i(\omega\tau+\epsilon)}].$$

Expand now $\varphi_j(x_1^*, \cdots, x_n^*, 0)$ in Fourier series and express this as a power series in $e^{i(\omega\tau+\epsilon)}$. The coefficient of the first power in the expansion of the right-hand side of (20) as

a whole is

$$p_j = -\tfrac{1}{2}A \sum_{k=1}^{n} \rho_{jk}(i\omega, 0)C_k + \frac{1}{2\pi}\int_{\pi}^{+\pi} \varphi_j(x_1^*, \cdots, x_n^*, 0)e^{-iv}dv, \quad (22)$$

where we have put under the sign of integral

$$v = \omega\tau + \epsilon. \quad (23)$$

The coefficient of $e^{-(\omega\tau+\epsilon)}$ is the conjugate complex quantity $\bar{p}_j$. Assuming still that $\Delta_1(\pm iL\omega, 0) \neq 0$ we shall be able to determine all $y_k$ from (20) as periodic functions provided the simpler system

$$\sum_{k=1}^{n} f_{jk}(D', 0)y_k = p_j e^{i(\omega\tau+\epsilon)}, \qquad j = 1, \cdots, n$$

admits a particular solution of the form $y_k = l_k e^{i(\omega\tau+\epsilon)}$ and the analogous circumstance presents itself for the system with the conjugate complex right sides. Substitute the expression just written for $y_k$ in the preceding equations and suppress the multiplier $e^{i(\omega\tau+\epsilon)}$ on both sides; the matrix of the resulting set of linear algebraic equations

$$\sum_{k=1}^{n} f_{jk}(i\omega, 0)l_k = p_j, \qquad j = 1, \cdots, n \quad (24)$$

is of rank $n - 1$, so that

$$\sum_{j=1}^{n} F_{jm}(i\omega, 0)f_{jk}(i\omega, 0) = 0, \qquad k = 1, \cdots, n.$$

By means of these relations we can, in view of the assumption (6) express the $l$-th row of $f(i\omega, 0)$ as a linear combination of the $n - 1$ remaining rows. In order that the equations (24) may be compatible, their matrix must retain the same rank $n - 1$ after it has been augmented by the column of the quantities $p_j$, and therefore

$$\sum_{j=1}^{n} F_{jm}(i\omega, 0)p_j = 0. \quad (25)$$

The differential equations with the conjugate complex right

sides would lead to a conjugate condition. Separating in real and imaginary parts we obtain two conditions involving exactly two as yet indeterminate constants $A$ and $C$.

5. *Determination of the Generating Amplitudes and of the First Frequency Correction.* The substitution for $p_j$ from (22) in the relation (25) gives

$$A \sum_{j=1}^{n} \sum_{k=1}^{n} F_{jm}(i\omega, 0) \rho_{jk}(i\omega, 0) C_k = \frac{1}{\pi} \sum_{j=1}^{n} F_{jm}(i\omega, 0) \int_{-\pi}^{+\pi} \varphi_j(x_1^*, \cdots, x_n^*, 0) e^{-iv} dv. \quad (26)$$

The co-factors of the corresponding elements of the $k$-th and of the $m$-th columns in the vanishing determinant $\Delta(i\omega, 0)$ being proportional, we can write

$$F_{jk}(i\omega, 0) = F_{jm}(i\omega, 0) \frac{F_{1k}(i\omega, 0)}{F_{1m}(i\omega, 0)};$$

hence, using the expression (7) for $C_k$

$$sF_{jk}(i\omega, 0) = F_{jm}(i\omega, 0) C_k. \quad (27)$$

On substitution of the expressions (21) the left hand side of the (26) therefore takes the form

$$As \sum_{j=1}^{n} \sum_{k=1}^{n} \left[ F_{jk}(D, \mu) \frac{\partial f_{jk}(D, \mu)}{\partial D} D \frac{d\alpha(\mu)}{d\mu} + F_{jk}(D, \mu) \frac{\partial f_{jk}(D, \mu)}{\partial \mu} \right]_{\substack{D=i\omega \\ \mu=0}}.$$

Then by the use of the rule for the differentiation of determinants and of the second expression (18) we obtain in place of (26)

$$As \left[ \frac{\partial \Delta(D, \mu)}{\partial D} DC + \frac{\partial \Delta(D, \mu)}{\partial \mu} \right]_{\substack{D=i\omega \\ \mu=0}} = \frac{1}{\pi} \sum_{j=1}^{n} F_{jm}(i\omega, 0) \int_{-\pi}^{+\pi} \varphi_j(x_1^*, \cdots, x_n^*, 0) e^{-iv} dv. \quad (28)$$

Suppose now $A \neq 0$ to begin with; writing down the conjugate condition it is seen that both relations are linear in $C$ and have the form

$$\eta C + \zeta = 0, \qquad \bar{\eta} C + \bar{\zeta} = 0$$

with $\eta \neq 0$. In order that these may be compatible we must have

$$\eta\bar{\zeta} - \bar{\eta}\zeta = -2iTm(\bar{\eta}\zeta) = -2iRe(-i\bar{\eta}\zeta) = 0.$$

The detailed form of this condition is

$$Re\, s\left[\frac{\partial\Delta(D,\mu)}{\partial D}\right]_{\substack{D=-i\omega \\ \mu=0}} \left\{ As\left[\frac{\partial\Delta(D,\mu)}{\partial\mu}\right]_{\substack{D=i\omega \\ \mu=0}} - \frac{1}{\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\tau}^{+\tau} \varphi_j(x_1^*, \cdots, x_n^*, 0)e^{-iv}dv \right\} = 0$$

or

$$rA = H(A), \tag{29}$$

where

$$r = |s|^2 Re\left[\frac{\partial\Delta(D,\mu)}{\partial D}\right]_{\substack{D=-i\omega \\ \mu=0}} \left[\frac{\partial\Delta(D,\mu)}{\partial\mu}\right]_{\substack{D=i\omega, \\ \mu=0}} \tag{30}$$

$$H(A) = \frac{1}{\pi} Re\, s\left[\frac{\partial\Delta(D,\mu)}{\partial D}\right]_{\substack{D=-i\omega \\ \mu=0}} \times \sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\tau}^{+\tau} \varphi_j(x_1^*, \cdots, x_n^*, 0)e^{-iv}dv. \tag{31}$$

The equation (29) is in general non-linear and yields one or more isolated values for the amplitude $A$ of the generating periodic solution. It is seen that in general, as a consequence of the presence of the non-linear terms in the differential equations, only few of the continuous multiplicity of periodic solutions of the simplified system can produce in their vicinity such solutions for the pseudo-linear system. The generating amplitudes being determined, (28) yields the corresponding values of the first frequency correction $C$. The equation (29) can be solved graphically as it is shown in the Fig. 1. To this effect we must find the points of intersection of the straight

line drawn through the origin and of the curve, which represent the left and the right hand sides of (29).

If now $A = 0$, which was formerly excluded, the condition (28) is always satisfied since $\varphi_j(0, \cdots, 0, 0)$ are constant and the integrals on the right vanish. We will however not indulge in a detailed discussion of this special case considered by E. Picard.[6]

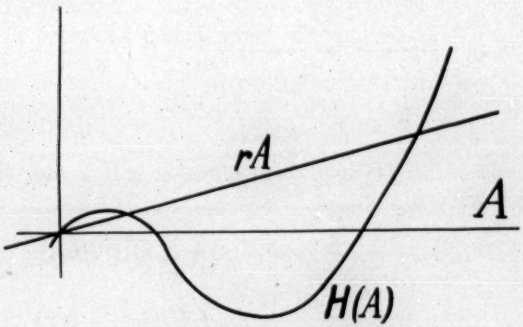



FIG. 1. Graphical determination of the generating amplitudes.

The periodicity condition being satisfied we can determine all unknowns $y_k$ from (20) as periodic functions:

$$y_k = \tfrac{1}{2}B[C_k e^{i(\omega\tau+\epsilon)} + \bar{C}_k e^{-i(\omega\tau+\epsilon)}] + \frac{i}{2}B'[C_k e^{i(\omega\tau+\epsilon)} - \bar{C}_k e^{-i(\omega\tau+\epsilon)}] + y_k^*. \quad (32)$$

$B$ and $B'$ are the constants of integration, the functions $y_k^*$ form a particular solution of (20). The constant $B$ will be determined from the second approximation only, $B'$ remains an indeterminate. It follows that only the generating solution (11) with the values of $A$ determined from the periodicity conditions can be used to serve as the first approximation for $x_k$. For $\alpha(\mu)$ we have the approximate expression

$$\alpha(\mu) \approx C\mu. \quad (33)$$

[6] E. Picard, Traité d'Analyse, Tome III, pp. 181–186, 3-me éd., Paris, 1928.

In practice we may often content ourselves with this first approximation as it is the case with the special problem considered in the 7-th and following paragraphs below, to which the reader may pass immediately if he wishes.

*6. Further Approximations.* Denote

$$z_k = \left(\frac{\partial^2 x_k}{\partial \mu^2}\right)_{\mu=0}, \quad k = 1, \cdots, n; \qquad c = \left(\frac{d^2\alpha}{d\mu^2}\right)_{\mu=0}, \tag{34}$$

$$\sigma_{jk}(D, \mu) = \frac{\partial f_{jk}(D, \mu)}{\partial D} D\frac{d^2\alpha}{d\mu^2} + \frac{\partial^2 f_{jk}(D, \mu)}{\partial D^2} D^2\left[\frac{d\alpha(\mu)}{d\mu}\right]^2 + 2\frac{\partial^2 f_{jk}(D, \mu)}{\partial D \partial \mu} D\frac{d\alpha(\mu)}{d\mu} + \frac{\partial^2 f_{jk}(D, \mu)}{\partial \mu^2} \tag{35}$$

and differentiate (1) two times with respect to $\mu$ using (13). It follows for $\mu = 0$

$$\sum_{k=1}^{n} f_{jk}(D', 0)z_k = -2\sum_{k=1}^{n} \rho_{jk}(D', 0)y_k - \sum_{k=1}^{n} \sigma_{jk}(D', 0)x_k^* + 2\left(\sum_{k=1}^{n} \frac{\partial \varphi_j}{\partial x_k} y_k + \frac{\partial \varphi_j}{\partial \mu}\right) \tag{36}$$

with the convention that in all cases when the variables involved in $\varphi_j(x_1, \cdots, x_n, \mu)$ or their derivatives are not indicated explicitly they are to be replaced by $x_1^*, \cdots, x_n^*, 0$. By means of the expressions (9) and (32) for $x_k^*$ and $y_k$ the following formulae for the coefficients of $e^{i(\omega\tau+\epsilon)}$ in the Fourier expansions of the right-hand sides result:

$$\begin{aligned} q_j = &-(B + iB')\sum_{k=1}^{n} \rho_{jk}(i\omega, 0)C_k - \tfrac{1}{2}A\sum_{k=1}^{n} \rho_{jk}(i\omega, 0)C_k \\ &+ \frac{1}{\pi}\frac{B}{2}\int_{-\pi}^{+\pi} \sum_{k=1}^{n} \frac{\partial \varphi_j}{\partial x_k}(C_k e^{iv} + \bar{C}_k e^{-iv})e^{-iv}dv \\ &+ \frac{1}{\pi}\frac{iB'}{2}\int_{-\pi}^{+\pi} \sum_{k=1}^{n} \frac{\partial \varphi_j}{\partial x_k}(C_k e^{iv} - \bar{C}_k e^{-iv})e^{-iv}dv \\ &+ \frac{1}{\pi}\int_{-\pi}^{+\pi} \left\{\sum_{k=1}^{n}\left[\frac{\partial \varphi_j}{\partial x_k} - \rho_{jk}(D', 0)\right]y_k^* + \frac{\partial \varphi_j}{\partial \mu}\right\} e^{-iv}dv; \end{aligned} \tag{37}$$

the periodicity condition for the equations (36) is

$$\sum_{j=1}^{n} F_{jm}(i\omega, 0)q_j = 0. \tag{38}$$

Excluding as before the case $A = 0$ we can write under the sign of integral

$$C_k e^{iv} + \bar{C}_k e^{-iv} = 2\frac{\partial x_k^*}{\partial A},$$

$$C_k e^{iv} - \bar{C}_k e^{-iv} = -\frac{2i}{A}\frac{dx_k^*}{dv}.$$

Using (26) and (37) and integrating by parts we obtain then for the coefficient of $iB'$ in the periodicity condition the expression

$$-\sum_{j=1}^{n}\sum_{k=1}^{n} F_{jm}(i\omega, 0)\rho_{jk}(i\omega, 0)C_k$$

$$+\frac{1}{\pi}\left(-\frac{i}{A}\right)\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\pi}^{+\pi}\sum_{k=1}^{n}\frac{\partial\varphi_j}{\partial x_k}\frac{dx_k^*}{dv}e^{-iv}dv$$

$$= -\frac{1}{A\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\pi}^{+\pi}\varphi_j e^{-iv}dv$$

$$-\frac{i}{A\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\left(\varphi_j e^{-iv}\Big|_{-\pi}^{+\pi} + i\int_{-\pi}^{+\pi}\varphi_j e^{-iv}dv\right),$$

which is zero since the periodic function $\varphi_j(x_1^*, \cdots, x_n^*, 0)e^{-iv}$ assumes equal values for $v = -\pi$ and $v = +\pi$. Thus the constant $B'$ does not enter in the periodicity condition and may be chosen arbitrarily.

The coefficients $q'$ and $q''$ of $B$ and of the quantity $c$ contained in $\sigma_{jk}(i\omega, 0)$ are determined in view of (21), (27) and (35) as

$$q' = -\sum_{j=1}^{n}\sum_{k=1}^{n} F_{jm}(i\omega, 0)\rho_{jk}(i\omega, 0)C_k$$

$$+\frac{1}{\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\pi}^{+\pi}\sum_{k=1}^{n}\frac{\partial\varphi_j}{\partial x_k}\frac{\partial x_k^*}{\partial A}e^{-iv}dv,$$

$$q'' = -\tfrac{1}{2}A\sum_{j=1}^{n}\sum_{k=1}^{n}\left[F_{jm}(D, \mu)\frac{\partial f_{jk}(D, \mu)}{\partial D}D\right]_{\substack{D=i\omega \\ \mu=0}} C_k,$$

or

$$q' = \frac{d}{dA}\left\{ - As\left[\frac{\partial\Delta(D,\mu)}{\partial D} DC + \frac{\partial\Delta(D,\mu)}{\partial\mu}\right]_{\substack{D=i\omega\\ \mu=0}} + \frac{1}{\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\pi}^{+\pi}\varphi_j e^{-iv}dv\right\}, \quad (39)$$

$$q'' = -\tfrac{1}{2}As\left[\frac{\partial\Delta(D,\mu)}{\partial D} D\right]_{\substack{D=i\omega\\ \mu=0}}. \quad (40)$$

With these coefficients the periodicity condition and the conjugate relation can be abbreviated to

$$q'B + q''c + \cdots = 0, \qquad q'B + q''c + \cdots = 0, \quad (41)$$

the unwritten terms being independent on $B$ and $c$. In order that these equations can be solved with respect to $B$ and $c$ their determinant must be other than zero:

$$q'\bar{q}'' - \bar{q}'q'' = 2iTm(q'\bar{q}'') = 2iRe(-iq'' \cdot q') \neq 0.$$

Or on substitution from (39) and (40) and reduction

$$Re\bar{s}\left[\frac{\partial\Delta(D,\mu)}{\partial D}\right]_{\substack{D=-i\omega\\ \mu=0}} \frac{d}{dA}\left\{ - As\frac{\partial\Delta(D,\mu)}{\partial D} DC + \frac{\partial\Delta(D,\mu)}{\partial\mu}\right]_{\substack{D=i\omega\\ \mu=0}} + \frac{1}{\pi}\sum_{j=1}^{n} F_{jm}(i\omega, 0)\int_{-\pi}^{+\pi}\varphi_j e^{-iv}dv\right\} \neq 0.$$

If we note besides that the quantity

$$\bar{s}\left[\frac{\partial\Delta(D,\mu)}{\partial D}\right]_{\substack{D=-i\omega\\ \mu=0}} s\left[\frac{\partial\Delta(D,\mu)}{\partial D} D\right]_{\substack{D=+i\omega\\ \mu=0}}$$

is purely imaginary, then it follows by (30) and (31)

$$\frac{d}{dA}\left[rA - H(A)\right] \neq 0. \quad (42)$$

This condition means that the straight line in the Fig. 1 must not be a tangent to the curve. If it is satisfied, we can determine $B$ and $c$ from (41) and find from (36) periodic expressions for $z_k$ with two constants of integration $C$ and $C'$. The

corresponding approximate formulae are

$$x_k = x_k^* + \frac{1}{1!} y_k \mu, \qquad \alpha = \frac{1}{1!} C\mu + \frac{1}{2!} c\mu^2. \tag{43}$$

In the case when $f_{jk}(D, \mu)$ and $\varphi_j(x_1, \cdots, x_n, \mu)$ are analytic in $x_1, \cdots, x_n, \mu$ this calculation may be continued indefinitely. The coefficients of $C$, $C'$ and of

$$\partial = \left(\frac{d^3\alpha}{d\mu^3}\right)_{\mu=0} \tag{44}$$

in the third periodicity condition coincide up to the numerical multipliers with the corresponding coefficients in the second condition. Similarly for further approximations. The resulting periodic solution

$$x_k = x_k^* + \frac{1}{1!} y_k \mu + \frac{1}{2!} z_k \mu^2 + \cdots \tag{45}$$

contains therefore an infinite sequence of indeterminate quantities $B'$, $C'$, $\cdots$ subject merely to the condition that for small values of $\mu$ the series

$$\frac{1}{1!} B'\mu + \frac{1}{2!} C'\mu^2 + \cdots$$

must converge. These quantities form a phase constant of the kind mentioned in par. 3. It can be evaluated from a suitable initial condition. The inequality (42) suffices to secure the complete determination of all other constants $B$, $C$, $\cdots$ and frequency corrections $c$, $\partial$, $\cdots$. The convergence of the series (45) as well as of the series

$$\alpha = \frac{1}{1!} C\mu + \frac{1}{2!} c\mu^2 + \cdots \tag{46}$$

for sufficiently small values of $\mu$ and with it the actual existence of the formally computed solution follow from the general theorems of H. Poincaré and W. D. MacMillan.[7] We may still point out that the determination of the generating amplitudes $A$ is quite independent of these series. At the same time the knowledge of the number and distribution of the

[7] Trans. of the American Mathem. Society 13 (1912), pp. 146–158.

generating solutions, from which the periodic solutions of the pseudo-linear system arise, is in all cases of a great use.

7. *The Differential Equations of a Follow-up System.* Under a follow-up system is meant a contrivance which makes a certain "follow-up" axis reproducing with minimum lag all angular displacements of another "leading" axis mechanically independent of the first. Such systems are employed, for instance, to support the bearings of the sensitive element of a gyro-compass [8] in order to diminish the positional and frictional forces opposing the displacements of the element relative to the bearings.

Let $\alpha$ and $\beta$ denote the angles of rotation of the follow-up and of the leading axes. Always as the "disagreement"

$$\psi = \alpha - \beta \tag{47}$$

exceeds a certain small upper limit $\frac{r}{2}$ an electric contact is made which causes a reversing motor to rotate the follow-up axis in such a sense that the disagreement would diminish. As it is known from experience the "hunting" of the follow-up system after the leading axis is always accompanied by self-excited angular oscillations the theory of which is the object of the following pages.

Various schemes of electric connections may be used, but to account in the first place for the essence of the phenomena we consider the simplest arrangement as shown in the Fig. 2. Two sets of contactor plates carried by the follow-up system are connected respectively with the armature $M$ of the motor and with the direct current generator $G$. The field coils of the motor are excited independently. To control the contactor system a rotating arm attached to the leading axis is employed.

The mechanical equation of the rotation of the armature is

$$\Theta\ddot{\alpha} = aI, \tag{48}$$

$\Theta$ denoting the moment of inertia referred to the follow-up axis, $I$—the current through the armature, $a$—a constant coefficient. The equation of the balance of electromotive forces

[8] Cf. Ervin S. Ferry, Applied Gyrodynamics, New York, 1933, p. 202.

may be written in the form

$$-F - L\frac{dI}{dt} - c\dot{\alpha} = RI, \tag{49}$$

$F$ denoting the applied electromotive force, $L$—the self-induction, $c$—the coefficient of the back electromotive force, $R$—the resistance. If the absolute value of $\psi$ does not exceed $\frac{\tau}{2}$ the current is cut off the motor and $F = 0$. For greater dis-

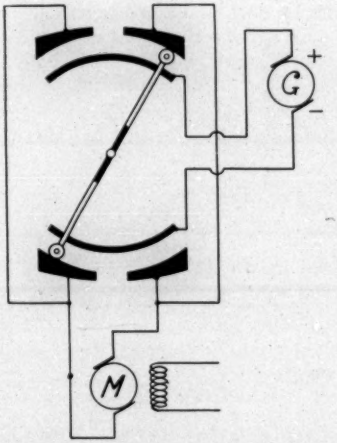



FIG. 2. Electric connections of the follow-up system.

agreement the electric circuit through the armature is completed in one or in the other sense. It follows that $F$ as a function of $\psi$ can be represented by such graph as shown in the Fig. 3.

Substituting for $\alpha$ from (47) and using the operator $D$ we have in place of the equations (48) and (49)

$$\begin{aligned} \Theta D^2\psi - aI &= -\Theta D^2\beta, \\ cD\psi + F(\psi) + CD\beta + (LD + R)I &= 0. \end{aligned} \tag{50}$$

We will restrict our investigation to the case of free vibrations governed by the equations which are obtained by cancelling both terms with $D\beta$ and $D^2\beta$. The solution of this simplified problem is sufficient for such a case as that of the follow-up system of a gyro-compass where the motion of the leading axis is very slow in comparison with the rotation of the follow-up axis by full speed of the motor.

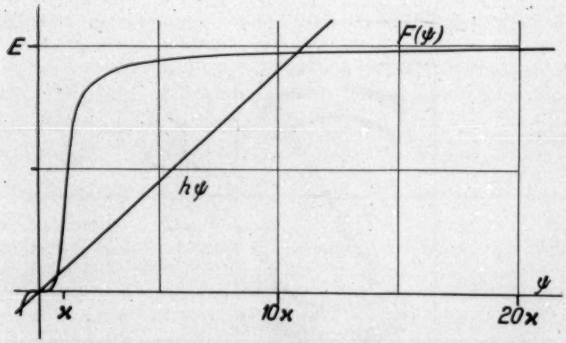



FIG. 3. The characteristic function of the follow-up motor.

Introducing a small parameter $\mu$ we shall consider the modified system

$$\Theta D^2\psi - aI = 0,$$

$$(cD + h)\psi + (LD + R)I = \frac{\mu}{\mu_1}[h\psi - F(\psi)], \qquad (51)$$

which coincides with the original set of equations of the free vibrations for the value $\mu_1$ of $\mu$; $h\psi$ is a linear approximation for the function $F(\psi)$ represented by the secant line in the Fig. 3. The constant $h$ may be referred to as "the middle slope" of the characteristic $F(\psi)$.

For $F(\psi) \equiv h\psi$ we have a linear system with the operational determinant

$$\Delta \equiv \begin{vmatrix} \Theta D^2, & -a \\ cD+h, & LD+R \end{vmatrix} \equiv \Theta LD^3 + \Theta RD^2 + acD + ah \qquad (52)$$

Of the Routh's conditions for this system only the following one

$$\Theta R \cdot ac - \Theta L \cdot ah > 0 \tag{53}$$

is of importance. All others are always satisfied since the coefficients $\Theta$, $L$, $R$, $a$, $c$, $h$ are positive. Let

$$L^* = \frac{Rc}{h}; \tag{54}$$

then we must have

$$L < L^*, \tag{55}$$

so that the damping condition leads here to the prescription of an upper boundary to the self-induction $L$.

When this limit is attained the characteristic equation can be by use of (54) written in the form

$$\Theta\frac{Rc}{h}D^3+\Theta RD^2+acD+ah\equiv\frac{\Theta Rc}{h}\left(D+\frac{h}{c}\right)(D^2+\omega^2)=0$$

with

$$\omega^2 = \frac{ah}{\Theta R}. \tag{56}$$

One root is real negative, two others are $\pm i\omega$. To this pair of purely imaginary conjugate roots corresponds a periodic solution $\psi^*$, $I^*$ of the linear system; assuming for $\psi^*$ the expression

$$\psi^* = A \sin \omega t \tag{57}$$

we have from the first equation (51) written for $\psi^*$, $I^*$:

$$aI^* = \Theta\ddot{\psi}^* = -\Theta A\omega^2 \sin \omega t$$

and

$$I^* = -\frac{h}{R} A \sin \omega t. \tag{58}$$

In both expressions (57) and (58) we may write also $\omega t + \epsilon$ instead of $\omega t$.

Return now to the pseudo-linear system (51) and consider the self-induction $L$ as a function of $\mu$ defined by the formula

$$L = L^*\left(1 - w\frac{\mu}{\mu_1}\right). \tag{59}$$

It follows

$$w\frac{\mu}{\mu_1} = \frac{L^* - L}{L^*}, \tag{60}$$

so that the constant $w$ represents the relative variation of the self-induction for $\mu = \mu_1$.

The system of equations (51) is now reduced to the exact form assumed in the general theory above. The non-linear corrections appearing in the right-hand sides with the factor $\mu$ are

$$\varphi_1 \equiv 0, \qquad \varphi_2 \equiv \frac{h\psi - F(\psi)}{\mu_1}. \tag{61}$$

In order that $\varphi_2$ remains finite, $\mu_1$ must not be less than of the order of $h\psi - F(\psi)/h$, so that, for instance, for a given $h\psi - F(\psi)$ the passage to the limit $\mu_1 \to 0$ must not be undertaken.

*8. Application of the General Formula.* We must now effect the calculations necessary for the application of the equation (28). Deriving the expression (52) we have by (54), (56) and (59):

$$\left[\frac{\partial\Delta(D,\mu)}{\partial D}D\right]_{\substack{D=i\omega\\ \mu=0}} = -2a(h + ic\omega), \tag{62}$$

$$\left[\frac{\partial\Delta(D,\mu)}{\partial\mu}\right]_{\substack{D=i\omega\\ \mu=0}} = i\frac{acw\omega}{\mu_1}. \tag{63}$$

Next put $m = 1$; then of the co-factors $F_{jk}$ only

$$F_{21} = a \tag{64}$$

is required and it follows from (8) that $C_1 = L_1 + iM_1 = s$; the formula (11) written for $k = 1$ can be brought in accordance with (57) if we put $L_1 = 1$, $M_1 = 0$, $\epsilon = -\frac{\pi}{2}$ and therefore $s = 1$.

The principal equation (28) assumes the form

$$Aa\left[-2hC + icw\left(-2C + \frac{w}{\mu_1}\right)\right] = \frac{a}{\pi}\int_{-\pi}^{+\pi}\varphi_2(A\sin u)e^{-iu}e^{i\pi/2}du$$

with

$$u = \omega\tau.$$

On separation in the real and imaginary parts we obtain two periodicity conditions

$$-2hAC = \frac{1}{\pi}\int_{-\pi}^{+\pi} \varphi_2(A \sin u) \sin u du,$$

$$c\omega A\left(-2C + \frac{w}{\mu_1}\right) = \frac{1}{\pi}\int_{-\pi}^{+\pi} \varphi_2(A \sin u) \cos u du$$

in real form. Next put

$$\chi(\psi) = \int_0^{\psi} \varphi_2(\psi) d\psi$$

and observe that

$$\int_{-\pi}^{+\pi} \varphi_2(A \sin u) \cos u du = \frac{1}{A}\int_{-\pi}^{+\pi} \varphi_2(A \sin u) d(A \sin u)$$

$$= \frac{1}{A}\chi(A \sin u)\Big|_{-\pi}^{+\pi} = 0;$$

the second periodicity condition then yields for the first frequency correction $C$ the formula

$$C = \frac{w}{2\mu_1}, \tag{65}$$

which is independent of $A$. Introducing this expression in the first periodicity condition and substituting for $\varphi_2$ from (61) we obtain

$$-\frac{hw}{\mu_1} A = \frac{1}{\pi\mu_1}\left[ hA\pi - \int_{-\pi}^{+\pi} F(A \sin u) \sin u du \right]$$

or

$$-hwA = hA - \frac{4}{\pi} J(A) \tag{66}$$

with the notation

$$J(A) = \frac{1}{4}\int_{-\pi}^{+\pi} F(A \sin u) \sin u du. \tag{67}$$

From the graph in the Fig. 3 it is seen that $F'(\psi) > 0$ and that when $\psi$ increases from 0 upwards $F''(\psi)$ is at first positive

then negative. We may note also that $F(\psi)$ and $F''(\psi)$ are odd, $F'(\psi)$ being an even function. The expression (67) and

$$J'(A) = \frac{1}{4}\int_{-\pi}^{+\pi} F'(A \sin u) \sin^2 u du, \tag{68}$$

$$J''(A) = \frac{1}{4}\int_{-\pi}^{+\pi} F''(A \sin u) \sin^3 u du \tag{69}$$

show that the same statements may be made in respect of $J(A)$, $J'(A)$, $J''(A)$ so that $J(A)$ can be represented by such a graph as given in the Fig. 4.

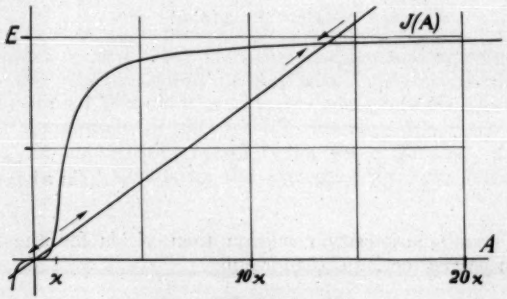



FIG. 4. The function $J(A)$.

The equation (66) can be solved graphically on the basis of example of the Fig. 1. If this equation would be expressed in the following final form

$$\boxed{J(A) = \frac{\pi}{4}(1 + w)hA}, \tag{70}$$

then the generating amplitudes can also be determined directly from the Fig. 4. They are namely the abscissae of the points of intersection of the curve $J(A)$ with a straight line drawn through the origin. The slope of this line is $\frac{\pi}{4}(1 + w)h$; according to the value of $w$ there may be for $A > 0$ two or no such points of intersection at all.

9. *The Stability Problem.* In order to test the stability of the found periodic oscillation with the coordinates $\psi$, $I$ we

consider a neighbouring slightly disturbed oscillation $\psi + \delta\psi$, $I + \delta I$. Substitute the modified coordinates for $\psi$, $I$ in the equations (51) and expand $F(\psi + \delta\psi)$ in the Taylor's series. All terms independent of $\delta\psi$, $\delta I$ disappear in consequence of the original equations being satisfied identically by $\psi$ and $I$. Neglecting the terms of the second and higher orders in $\delta\psi$, $\delta I$ we obtain two linear equations in variations

$$\begin{aligned} \Theta D^2\delta\psi - a\delta I &= 0, \\ (cD + h + h_1)\delta\psi + (LD + R)\delta I &= 0 \end{aligned} \qquad (71)$$

with

$$h_1 = \frac{\mu}{\mu_1}[F'(\psi) - h]. \qquad (72)$$

A complete investigation of the integrals of these would not be easy and would require a study of certain as yet not fully known properties of the systems of linear equations with periodic coefficients. We restrict ourselves therefore to an approximate treatment which will show that for small values of $\mu$ the stability or instability can be read from the Fig. 4 in the same way, as, say, in the theory of regulation of steam engines. We content ourselves namely with the constant mean value of the coefficient $h_1$ containing the small factor $\mu$ and in computing this we replace $F'(\psi)$ by the approximate expression $F'(A \sin \omega\tau)$. The error arising from such a substitution can be considerable at such points where $F'(\psi)$ varies rapidly; but we may expect that in passing to the mean value the error would be reduced. It is

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi} F'(A \sin u)du = \frac{1}{2\pi}\int_{-\pi}^{+\pi} F'(A \sin u)(\cos^2 u + \sin^2 u)du$$

$$= \frac{1}{2\pi}\left[\frac{1}{A}\int_{-\pi}^{+\pi} \cos u F'(A \sin u)d(A \sin u)\right.$$

$$\left. + \frac{d}{dA}\int_{-\pi}^{+\pi} F(A \sin u) \sin u du\right] = \frac{1}{2\pi}\left[\frac{1}{A}\cos u F(A \sin u)\Big|_{-\pi}^{+\pi}\right.$$

$$\left. + \left(\frac{d}{dA} + \frac{1}{A}\right)\int_{-\pi}^{+\pi} F(A \sin u) \sin u du\right]$$

$$= \frac{2}{\pi A}\frac{d}{dA}[AJ(A]$$

and therefore

$$h_1 \approx \frac{\mu}{\mu_1}\left\{\frac{2}{\pi A}\frac{d}{dA}[AJ(A)] - h\right\}$$

$$= \frac{\mu}{\mu_1}\frac{1}{2A}\frac{d}{dA}A\left[\frac{4}{\pi}J(A) - hA\right]. \quad (73)$$

With positive parameters $\Theta$, $L$, $R$, $a$, $c$ only two of the Routh's conditions for the system of equations (71) having now all their coefficients constant are to be considered and these can be written in the forms

$$h + h_1 > 0, \qquad Rc > L(h + h_1). \quad (74)$$

On substituting for $h_1$ the expression just found, we express the first inequality as

$$2Ah + \frac{\mu}{\mu_1}\frac{d}{dA}A\left[\frac{4}{\pi}J(A) - hA\right]$$

$$\equiv 2Ah\left(1 - \frac{\mu}{\mu_1}\right) + \frac{4}{\pi}\frac{\mu}{\mu_1}\frac{d}{dA}AJ(A) > 0.$$

As it was shown above $J(A)$ is a monotonic increasing function; it follows that for positive values of $A$, $h$ and $0 \leqq \mu \leqq \mu_1$ the preceding condition is always satisfied.

The second inequality (74) can be expressed with the aid of (73) as

$$(Rc - Lh)2A - L\frac{\mu}{\mu_1}\frac{d}{dA}A\left[\frac{4}{\pi}J(A) - hA\right]$$

$$= \frac{d}{dA}A\left\{(Rc - Lh)A - L\frac{\mu}{\mu_1}\right]\frac{4}{\pi}J(A) - hA\Big] > 0$$

or, using (54) and (59) for small values of $\mu$,

$$\frac{d}{dA}A\left\{\left[Rc - \frac{Rc}{h}\left(1 - w\frac{\mu}{\mu_1}\right)h\right]A - \frac{Rc}{h}\frac{\mu}{\mu_1}\left[\frac{4}{\pi}J(A)\right.\right.$$

$$\left.\left. - hA\right]\right\} = \frac{d}{dA}A\left[(1 + w)hA - \frac{4}{\pi}J(A)\right]\frac{Rc}{h}\frac{\mu}{\mu_1} > 0.$$

In view of the equation (66) satisfied by $A$ this can be

written in the form

$$\boxed{\frac{dJ(A)}{dA} < \frac{\pi}{4}(1+w)h} \tag{75}$$

This inequality will be satisfied for such points of intersection of the curve with the straight line in the Fig. 4, where the latter passes from the lower to the upper of the two regions into which the right half-plane is divided by the curve. In this case the periodic solution of the pseudo-linear system arising from the corresponding generating solution will be stable, in the other case—unstable. And namely, of the two points of intersection in the Fig. 4 that on the left hand ($A = A_1$) corresponds to the unstable and that on the right ($A = A_2$)—to the stable self-excited steady oscillations. This is marked in the figure by arrows.

The physical meaning of the results just obtained is that only very small disturbances of the system can lead to subsiding oscillations tending to come to rest. Moderate initial disturbances which may always occur in practice produce oscillations growing to the constant limiting amplitude $A_2$. Such steady oscillations of the follow-up systems of the gyrocompasses are well known and considered as useful since they lead to an additional reduction of the effect of friction in the bearings. By larger initial disturbances the follow-up motor does its part and the oscillations are damped but only to the same limiting amplitude $A_2$.

*10. Analytical Representation of the Characteristic Curve of the Follow-up Motor.* For numerical computations we may apply characteristic curves obtained experimentally and determine $J(A)$ by means of a numerical integration.

An appropriate analytical expression can also be of use. For instance, let

$$F(\psi) = \frac{E}{\pi}\left[\frac{q}{2p}\ln\frac{(\psi-p)^2+q^2}{(\psi+p)^2+q^2} + \operatorname{arctg}\frac{\psi-p}{q} + \operatorname{arctg}\frac{\psi+p}{q}\right], \tag{76}$$

where $E$, $p$, $q$ are positive coefficients; it follows that

$$F(0) = 0, \qquad F(\pm\infty) = \pm E,$$
$$F'(\psi) = \frac{4qE\psi^2}{\pi[(\psi+p)^2+q^2][(\psi-p)^2+q^2]}. \tag{77}$$

Putting

$$p = \sqrt{\frac{\varkappa^2 + \varkappa\sqrt{\varkappa^2 + \lambda^2}}{2}}, \qquad q = \sqrt{\frac{-\varkappa^2 + \varkappa\sqrt{\varkappa^2 + \lambda^2}}{2}} \tag{78}$$

we obtain

$$F'(\psi) = \frac{4qE\psi^2}{\pi[(\psi^2 - \varkappa^2)^2 + \varkappa^2\lambda^2]},$$

$$F''(\psi) = \frac{8qE\psi[-\psi^4 + \varkappa^2(\varkappa^2 + \lambda^2)]}{\pi[(\psi^2 - \varkappa^2)^2 + \varkappa^2\lambda^2]^2}, \tag{79}$$

$$F'(0) = F'(\pm\infty) = 0, \qquad F'(\varkappa) = \frac{4qE}{\pi\lambda^2}. \tag{80}$$

$F(\psi)$ is therefore an odd increasing function, vanishing with $\psi$ and tending to finite limiting values for $\psi = \pm\infty$. Its derivative $F'(\psi)$ is even and also vanishes with $\psi$; it grows with $\psi$ for small positive values of this variable, attains a maximum value $F'(\psi^*)$ at a certain point $\psi = \psi^*$, then decreases and tends to zero when $\psi \to \infty$. It follows from (79) that

$$\psi^* = \sqrt[4]{\varkappa^2(\varkappa^2 + \lambda^2)}, \qquad F'(\psi^*) = \frac{2qE}{\pi\varkappa(\sqrt{\varkappa^2 + \lambda^2} - \varkappa)}. \tag{81}$$

If $\lambda$ is small the point $\psi = \psi^*$ is very near to $\psi = \varkappa$; the function $F(\psi)$ increases in this place very quickly. All these circumstances accord with the run of the characteristic curve as described above (par. 8).

The coefficient $h$ of the linear approximation can be determined from the equation for mean values

$$\int_0^l h\psi d\psi = \int_0^l F(\psi)d\psi,$$

where $(0, l)$ is the working region of the motor. By means of an integration by parts this equation can be written in the form

$$h = \frac{2}{l^2}\left[ lF(l) - \int_0^l \psi F'(\psi)d\psi \right], \tag{82}$$

where, by the first formula (79),

$$\int_0^l \psi F'(\psi)d\psi = \frac{2qE}{\pi}\left[\frac{1}{2}\ln\frac{(l^2-x^2)^2+x^2\lambda^2}{x^2(x^2+\lambda^2)} + \frac{x}{\lambda}\left(arctg\frac{l^2-x^2}{x\lambda}+arctg\frac{x}{\lambda}\right)\right]. \quad (83)$$

An integration by parts can also be applied to the expression (67) for $J(A)$; this gives

$$J(A) = A\int_0^{\frac{\pi}{2}} F'(A\sin u)\cos^2 u du$$

or, by (79) and using the substitution $x = tgu$,

$$J(A) = \frac{4qEA^3}{\pi}\int_0^\infty \frac{x^2dx}{\{[A^2x^2-x^2(x^2+1)]^2+x^2\lambda^2(x^2+1)^2\}(x^2+1)}.$$

Next put

$$\gamma(A) = \sqrt{\frac{x^2A^2-x^2(x^2+\lambda^2)+\sqrt{x^2(x^2+\lambda^2)[(A^2-x^2)^2+x^2\lambda^2]}}{2[(A^2-x^2)^2+x^2\lambda^2]}},$$

$$\delta(A) = \sqrt{\frac{-x^2A^2+x^2(x^2+\lambda^2)+\sqrt{x^2(x^2+\lambda^2)[(A^2-x^2)^2+x^2\lambda^2]}}{2[(A^2-x^2)^2+x^2\lambda^2]}}, \quad (84)$$

and note that

$$[(A^2-x^2)x^2-x^2]^2+x^2\lambda^2(x^2+1)^2 = [(A^2-x^2)^2+x^2\lambda^2][(x-\gamma)^2+\delta^2][(x+\gamma)^2+\delta^2].$$

We have then the following expansion in partial fractions

$$\frac{x^2}{\{[(A^2-x^2)x^2-x^2]^2+x^2\lambda^2(x^2+1)^2\}(x^2+1)} = \frac{1}{A^4}\left\{-\frac{1}{x^2+1}+\frac{(1-\gamma^2-\delta^2)(x-\gamma)+\gamma(1+\gamma^2+\delta^2)}{4\gamma[(x-\gamma)^2+\delta^2]} + \frac{-(1-\gamma^2-\delta^2)(x+\gamma)+\gamma(1+\gamma^2\to\delta^2)}{4\gamma[(x+\gamma)^2+\delta^2]}\right\}$$

and by means of this

$$J(A) = Eq \frac{1 - 2\delta(A) + \gamma^2(A) + \delta^2(A)}{A\delta(A)}, \tag{85}$$

$$J(0) = 0, \qquad J(\pm \infty) = \pm E. \tag{86}$$

The curves in the Figs. 3 and 4 are computed and plotted exactly with the formulae (76), (78), (84) and (85) for $\lambda^2 = 0.21\varkappa^2$. At first sight both curves can hardly be distinguished from one another. The scale units for the two coordinate axes in these diagrams are chosen to be $\varkappa$ and $E$ respectively.

# A NOTE ON THE SYMMETRY OF CERTAIN THERMODYNAMIC RELATIONS.

JOSEPH W. CREELY, GEORGE C. LE COMPTE AND WALTER W. LUCASSE.

The thermodynamic state of a system can be expressed in various ways by means of a number of fundamental variables. Excluding from consideration magnetic, surface, gravitational and certain other phenomena, the variables which will determine the state of the system may be listed as the heat content, the intrinsic energy, the work content, the free energy, temperature, pressure, entropy, volume, the heat and work. Using the respective symbols commonly employed by American chemists, these quantities, for the sake of the present discussion, fall into three groups:

(1) $H$, $E$, $A$, $F$
(2) $T$, $P$, $S$, $V$
(3) $q$, $w$

The first group is made up of energy quantities, the values of which depend only upon the state of the system. Their changes in value depend only upon the difference between the initial and final state and are independent of the path. The first two are closely related, increase in these quantities being directly measured by the heat absorbed, the first if the process is carried out at constant pressure and the second if at constant volume. Since both are truly energy quantities, following Kamerlingh Onnes, the former is sometimes called the *enthalpy*, thus obviating the possible misconception inherent in the term *heat content* which, however, is still more commonly used in chemical thermodynamics.

The values of the second group of quantities depend, like those of the first, only upon the state and their differences are independent of the path. In themselves, these quantities are not energy functions, but they are immediately related to energy. Thus, the product of the temperature and the

entropy change, and the product of the pressure and the volume change, are both energy quantities.

Defining energy as work, or anything which can be produced from or converted into work, associates the quantities in the third group and suggests the possibility of interrelationships with the other two groups. However, the members of the last group are unique in the fact that the value in each case depends upon the path traversed in going from the initial to the final state. Thus $q$ is equal to $TdS$ only if the process is reversible; and only if no type of work is involved other than that of expansion and the path is restricted in the manner indicated above, is $q$ equal to $\Delta H$ or $\Delta E$. Likewise, the work done may be equal to or less than the energy decrease, $\Delta A$ or $\Delta F$ depending upon the conditions, only if the path is restricted and the process reversible. The work of expansion may be the maximum or indefinite in value depending upon the operation of the opposing force against which the increase in volume is taking place.

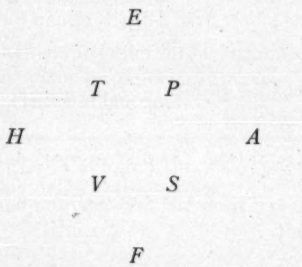
Two points become evident from this discussion of heat and work: relations between either of the quantities in the third group and those in the other two become equalities only in the limit; and, since $q$ and $w$ are dependent upon the path, their values do not express the state of the system. While changes in any of the ten fundamental variables will alter the thermodynamic state of the system, only the variables of the first two groups express the thermal and mechanical properties of the system under a given set of conditions. These two aspects are manifest in the statement of the First Law in the form:

$$\Delta E = q - w.$$

A quantity which is independent of the path and which therefore can express the state of the system at any point is here equated to the algebraic sum of the values of the heat and work. Only because of the compensation of the values does an equality obtain and only when the path is restricted does either $q$ or $w$ alone become equal to a quantity characteristic of the state.

In relating the thermodynamic quantities, if equalities are to result, it is essential to bear in mind the nature of the

quantities in the last group or to confine consideration to the first two groups in which the necessary limitations are imposed by definition. With this restriction, the defining equations of the first four quantities and the interrelationships of the first eight show numerous cases of symmetry, as the foregoing discussion of the groups might suggest. Such symmetry has frequently been indicated or implied in connection with particular equations and relationships. It is evident whenever Jacobians or Hessians are involved. The symmetry of some of the more elementary relations is strikingly manifest upon consideration of the following diagram:




Describing a series of ellipses from $H$ or $A$ through the diagonal members of the inner group, giving a negative sign to the inner product whenever the diagonal passage is from right to left, associates the defining equations of the energy quantities. Thus,

from $H$ through $T$, $S$ and $F$: $H = TS + F$,
from $H$ through $V$, $P$ and $E$: $H = VP + E$,
from $A$ through $P$, $V$ and $F$: $A = -PV + F$,
from $A$ through $S$, $T$ and $E$: $A = -ST + E$.

The symmetrical grouping can further be shown by considering the diagram in a somewhat different way to lead to the same equations. Passing clockwise from one energy quantity to the next, each quantity is equal to the next plus or minus the product of the parallel inner quantities, the sign being positive if the inner diagonal in the same direction is from left to right and negative if from right to

left. Thus,

$$H = E + VP,$$
$$E = A + TS,$$
$$A = F - PV,$$
$$F = H - ST.$$

The two relations involving the opposite energy quantities in the above diagram can readily be found by eliminating $E$ from the second and fourth equations immediately below the diagram, or $F$ from the first and third, and by eliminating $H$ from the first pair of equations, or $A$ from the second. The symmetry of the functions can be seen from the fact that these relations follow from substitution in the formal equation:

Each energy quantity = the opposite energy quantity ± (each adjacent inner quantity × its diagonal member)

in which the sign is

+ if the diagonal from the first energy quantity passes from left to right,
− if the diagonal from the first energy quantity passes from right to left.

Thus:

$$H = A + TS + VP,$$
$$E = F + TS - PV,$$
$$A = H - ST - PV,$$
$$F = E - ST + VP.$$

Although matter may not enter or leave a closed system, work may be done by or on the system and heat may be introduced or withdrawn. Changes in these quantities from the third group above result in changes in the characteristic energy functions. The effect on the energy quantities of an infinitesimal, reversible change in thermodynamic state may be summarized by the unifying equation:

$d$ (energy quantity) = ± (each adjacent inner quantity) $d$ (its diagonal member)

in which the sign is

+ if the diagonal from the energy quantity passes from left to right,
− if the diagonal from the energy quantity passes from right to left.

Thus:

$$dH = TdS + VdP,$$
$$dE = TdS - PdV,$$
$$dA = -SdT - PdV,$$
$$dF = -SdT + VdP.$$

Considering the partials of the energy quantities from this last set of equations leads to important relations for a closed system in which the only work involved, if any, is that of expansion. Thus,

$$\left(\frac{\partial E}{\partial S}\right)_V = T \quad \text{and} \quad \left(\frac{\partial E}{\partial V}\right)_S = -P.$$

Upon application of the commutative relationship from the calculus, the equations of Maxwell, involving only the inner quantities in the above diagram, result immediately. The following generalized equation shows the symmetry of the equations and of the placement of the quantities in the diagram. Passing lines through the inner pairs, parallel to the energy function under consideration,

$$\frac{\partial^2(\text{energy quantity})}{\partial(\text{farther inner quantity})\partial(X)} = \pm \frac{\partial(\text{nearer inner quantity})}{\partial(\text{farther inner quantity})_X},$$

in which the sign is

+ if the diagonal from the energy quantity passes from left to right,
− if the diagonal from the energy quantity passes from right to left, and $X$ is the member diagonal to the nearer inner quantity, that is, $X$ is the other farther inner quantity.

Thus:

$$\frac{\partial^2 H}{\partial P \partial S} = \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P,$$

$$\frac{\partial^2 E}{\partial S \partial V} = -\left(\frac{\partial P}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S,$$

$$\frac{\partial^2 A}{\partial V \partial T} = -\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial P}{\partial T}\right)_V,$$

$$\frac{\partial^2 F}{\partial T \partial P} = \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T.$$

The equations involving the inner quantities are due to Maxwell and are of extremely great importance not only in themselves since they permit the control of variables to those which are readily determined experimentally but also because they lead to the development of other significant relations.

It might be suggested that an additional qualification imposed upon the unifying equations above would reduce their generality to single cases. This is scarcely true, however, since the limitation which groups the resulting equations into pairs is one of sign only and the symmetry of the placement of the quantities persists throughout all of the equations. Furthermore, the restriction of sign in all of the cases considered is similar in nature. It should, perhaps, be further emphasized that the unifying "equations" throughout are formal only and not mathematical in the usual sense. They are neither derived from nor do they lead to other equations. They merely correlate the fundamental thermodynamic functions in a manner which stresses an underlying similarity of form.

The symmetry of certain thermodynamic relations which has been illustrated by several cases and emphasized by means of a diagram, might suggest such a scheme as a mnemonic. However, as with many memory aids, it might be found that more effort would be required to keep in mind the specific limitations than to memorize a few defining equations and apply to them sound mathematical reasoning. The picture presented, nevertheless, is not without value. It places stress upon the fundamental similarities and differences involved in the thermodynamic variables and their interrelationships.

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## NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

### NEW USES FOR BESSEMER STEEL.

The development of the open-hearth steel-making process caused the use of Bessemer steel to decrease to an almost negligible total a few decades ago. In competition with the superior open-hearth product, it was considered suitable for only a few specific uses, such as free-machining steel. Recently, however, the process has been considerably improved, and this fact, together with today's unprecedented demand for steel, has led to the consideration of Bessemer for many more purposes.

An important contribution of the American steel industry to the war effort is the steel landing mat for constructing airplane runways. By using these interlocking mats, which are stamped out of heavy-gage sheet steel, a pasture can be converted into an airfield in a few days. Advice on the use of Bessemer steel for this specific application was requested of the Bureau by the Army Engineer Corps. On the basis of present information, there appeared to be no reason whatsoever for withholding approval of Bessemer steel for this purpose, and a recommendation was made accordingly. As a matter of fact, this would seem to be an almost ideal use for a steel which requires in its preparation only a fraction of an hour, instead of the 8 or 10 hours necessary for all other processes.

### CATHODIC PROTECTION OF PIPE LINES.

Cathodic protection as a means of preventing corrosion of buried pipe lines has been widely used for a number of years and experience has shown that a properly designed and maintained system is both satisfactory and economical.

Until recently, little has been known about the current density necessary to prevent corrosion. The common practice has been to cause current to flow from a buried anode to

* Communicated by the Director.

the structure in sufficient quantities to insure an arbitrarily chosen difference of potential between the structure and a reference electrode, or to cause the potential to decrease by a chosen amount which experience has shown to be sufficient to prevent corrosion. It has been shown that the potential of buried steel with respect to a reference electrode in the adjacent soil depends on the aeration and pH value of the soil and that the application of current to a buried structure may gradually raise or lower its potential, depending on the reactions resulting from the flow of current. It follows that neither the potential of the buried structure nor a specified change in the potential necessarily indicates its condition with respect to corrosion.

If buried corroding steel is made a cathode, its true potential with respect to adjacent earth remains constant when the current flowing to the cathode is increased, until the local corrosion currents have been overcome. Application of additional current changes the potential of the cathode in proportion to the logarithm of the additional current. The current just sufficient to change the potential of the cathode is the current required to prevent its corrosion.

The potential of a cathode with respect to a reference electrode in the earth, as usually measured, includes a drop due to resistance in the circuit, and this must be eliminated or deducted if the critical current is to be recognized. Methods for accomplishing this have been worked out by K. H. Logan of the Bureau in coöperation with J. M. Pearson, I. A. Denison, R. F. Hadley, and A. V. Smith. The apparatus required is described in detail in a paper which has been submitted for publication in Petroleum Engineer. Results of using the new criterion for the current required to prevent corrosion are presented, and the results of using the old and new methods are compared.

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### EFFLORESCENCE ON PLASTER AND MASONRY.

Efflorescence occurs as a deposit of crystalline salts on the surface of plaster or masonry. The salts ordinarily are present in the interior of the wall and are dissolved by water, transported to the surface and there deposited by evaporation.

The complete results of a recent study of efflorescence on plaster and masonry, by Dana L. Bishop, will be found in the *Journal of Research* for May (RP1538). Magnesium sulfate, although not a constituent of plaster white coat or base coat, was found to be a constituent of efflorescence. Its formation is explained by the reaction of water and $CO_2$ with the white-coat plaster. Carbon dioxide converts the magnesia to slightly soluble $MgCO_3 \cdot 3H_2O$ which reacts with gypsum to form $MgSO_4$. Magnesium sulfate was produced experimentally by the action of $CO_2$ on a water suspension of MgO and $CaSO_4 2H_2O$. Hydrated magnesium sulfate efflorescence also was formed on white-coat plaster specimens containing dolomitic hydrated lime and gypsum when treated with moist $CO_2$ under pressure and then allowed to dry. The disintegration accompanying efflorescence is caused by the large volume changes involved in the formation and crystallization of $MgCO_3 \cdot 3H_2O$ and $MgSO_4 \cdot 7H_2O$.

Since moisture is necessary for the formation and transfer of these salts, plaster when kept dry does not effloresce.

$MgSO_4$ also is found as efflorescence on cement products, possibly being derived from MgO and sulfate in the cement by reaction with $CO_2$ and water. The compound $3CaO \cdot Al_2O_3 \cdot 3CaSO_4 \cdot 31H_2O$, a constituent of cement, was decomposed by $CO_2$ and water, making $CaSO_4$ available for reaction with $MgCO_3 \cdot 3H_2O$ to form soluble $MgSO_4$.

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### HYDRATED CALCIUM ALUMINATES.

The extensiveness of the literature that has grown around the hydrated calcium aluminates is proof of the scientific interest in and technological importance of the e compounds. Nevertheless, it was felt at the Bureau that a study should be made of the system $CaO\text{-}Al_2O_3\text{-}H_2O$, particularly as regards phase equilibria. This was not alone because of its direct bearing on the setting and hardening of high-alumina and portland cements, but also on account of possible applications in the fields of geology, soil equilibria, water purification, and the extraction of alumina from crude bauxite. Such a study was, therefore, undertaken by Lansing S. Wells, W. F. Clarke, and H. F. McMurdie. As a result, diagrams have been con-

structed showing the solubilities of the various phases at temperatures of 21° and 90° C. The following is an abstract of the report which will be published as RP1539 in the May number of the *Journal of Research*.

The solid phases were investigated by petrographical and X-ray diffraction methods. By means of the diffraction patterns, it was found that the so-called hexagonal tricalcium aluminate hydrate is in reality a mixture of hexagonal $2CaO \cdot Al_2O_3 \cdot 8H_2O$ and hexagonal $4CaO \cdot Al_2O_3 \cdot 13H_2O$ intercrystallized in equi-molecular proportions. On standing, dry dicalcium aluminate hydrate is slowly converted into the tetracalcium aluminate hydrate and hydrated alumina. Loss of water of hydration in the tetracalcium aluminate hydrate results in a decrease in the unit cell along the *c* axis.

Although the hexagonal di- and tetracalcium aluminate hydrates exist only as metastable phases in the system $CaO$-$Al_2O_3$-$H_2O$ between temperatures of 21° and 90° C., their approximate solubility relationships were ascertained at 21° C. There appears to be a metastable invariant point for these two solid phases and a solution containing between 0.5 and 0.6 g. of CaO per liter and 0.10 and 0.15 g. of $Al_2O_3$ per liter. As the temperature increases the hexagonal aluminates become less stable.

The only stable solid phases that occur in the system $CaO$-$Al_2O_3$-$H_2O$ over the temperature range from 21° to 90° C. are: Gibbsite, $Al_2O_3 \cdot 3H_2O$; the isometric tricalcium aluminate hexahydrate, $3CaO \cdot Al_2O_3 \cdot 6H_2O$; and $Ca(OH)_2$.

The solubility curves of gibbsite and isometric $3CaO \cdot Al_2O_3 \cdot 6H_2O$ in the system $CaO$-$Al_2O_3$-$H_2O$ were determined at 21° and 90° C. Over this temperature range, gibbsite is the stable phase up to a concentration of 0.33 g. of CaO per liter; at concentrations greater than this, the isometric hexahydrate is the stable phase until those points are reached at which $Ca(OH)_2$ also appears as a solid phase. The invariant points for gibbsite and the isometric phase occur at a concentration of 0.33 g. of CaO per liter, but with the concentration of $Al_2O_3$ increasing from 0.02 g. of $Al_2O_3$ per liter at 21° C. to 0.11 g. of $Al_2O_3$ per liter at 90° C.

### ELECTROMETRIC TITRATIONS.

A paper by Roger G. Bates, Gerda L. Siegel, and S. F. Acree in the May *Journal of Research* (RP1537) reports an improvement in the method of electrometrically titrating a weak monobasic acid to determine its dissociation constant and the pH values of buffer solutions prepared from the weak acid and its salts.

It is often desirable to determine pH values for a series of mixtures of a weak acid and its salts with precision greater than that obtainable by the usual practical pH assembly with liquid junction, which may give results in error by 0.02 to 0.05 pH unit. In principle, electrometric titration is well suited for a rapid determination of the pH changes in the neutralization of a weak acid. In practice, however, the limited accuracy of the results has been responsible for the gradual adoption of more precise methods for determining dissociation constants and pH values.

Two modifications have been suggested which improve the accuracy of the titration method and extend its usefulness. In the present paper, it is shown that an electrometric titration, performed (*a*) in a cell without liquid junction and (*b*) under conditions of unchanging ionic strength, is capable of yielding dissociation constants accurate to about 0.01 unit. The improved method was checked by a determination of the titration curves and dissociation constants of formic and acetic acids.

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### SIMPLIFIED PRACTICE RECOMMENDATION FOR HEAVY FORGED HAND TOOLS.

The War Production Board in 1942 requested the Bureau to work out a further simplification of the line of tools shown in Simplified Practice Recommendation R17-35 as a means of conserving critical metals, and making the most effective use of the limited supply.

As first promulgated in 1924, Recommendation R17 established a simplified list of weights, sizes, and types of forged tools that showed a reduction of almost 50 per cent. from the then existing variety. In 1927 the recommendation was revised by the addition of eye sizes and shapes, and in 1931 its

scope was further enlarged to include a simplified list of railroad and other industrial tools, as well as lengths for hammers, mauls, and sledges, only the weights of which had been previously given. A third edition, which became effective in 1935, covered mainly the addition of certain tools to meet requirements of the manufacturers' NRA code.

In July 1942, a proposed revision of Recommendation R17-35 was prepared in collaboration with the War Production Board and representatives of the manufacturers. This draft, with certain modifications, was approved by the Heavy Forged Hand Tool Industry Advisory Committee, and issued by the Board as Schedule IV to Limitation Order L-157, Hand Tools Simplification, to be effective November 3, 1942. To take care of some changes proposed by the manufacturers after the schedule went into effect, an amendment to Schedule IV was issued on February 25, 1943.

Under this schedule, 351 types and sizes of heavy forged hand tools are permitted to be manufactured. It covers tools that have heretofore been offered in approximately 1,170 varieties, of which 70 per cent. have been eliminated. Included in the 819 eliminated items are about 300 varieties of 78 tools that have been discontinued altogether.

This simplified list has also been approved as an emergency revision of Recommendation R17-35, which will be reissued as R17-43, Heavy Forged Hand Tools. The kinds of tools covered are as follows: Bars; blacksmiths' anvil tools; hammers, mauls, and sledges; hoes, mattocks; picks; railway track tools; tongs; wedges; and mine blasting tools, mine breast drills, and miscellaneous tools.

After the emergency the recommendation will again be subject to regular review for revision or reaffirmation by the industry's standing committee.

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### COMMERCIAL STANDARD FOR SCREW THREADS AND TAP-DRILL SIZES.

Under the title Screw Threads and Tap-Drill Sizes, Commercial Standard CS24-43, the Bureau of Standards has just released a second edition of American National Standard and Special Screw Threads, CS24-30 and CS25-30.

The revised standard is somewhat wider in scope than the previous editions of the two standards which it consolidates. It provides the essential specifications, definitions, symbols, and dimensional data on the predominating sizes and classes of fit of American National Screw Threads in the six following series: Coarse, fine, extra fine, 8-pitch, 12-pitch, and 16-pitch. Corresponding to these are six tables of stock tap drill-sizes, with dimensional data on diameters and depths of threads.

For the protection and guidance of the purchaser, the publication includes a recommended uniform wording of a statement to be incorporated in labels, invoices, catalogs, etc., guaranteeing that screw threads of the respective classes of fit conform to the standard.

The standard became effective for new production on February 10, 1943. Copies are available from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 10 cents each.

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### COMMERCIAL STANDARDS FOR WARM AIR FURNACES EQUIPPED WITH VAPORIZING POT-TYPE OIL BURNERS.

The Bureau has just released Commercial Standard (Emergency) E-CS104-43, entitled, "Warm Air Furnaces Equipped with Vaporizing Pot-Type Oil Burners," which has been accepted by the industry as its standard of practice, effective for new production from January 1, 1943.

The Standard records, for the benefit of all concerned, the performance requirements and methods of test for oil-burning furnaces with either gravity or forced air circulation. It includes construction and design requirements, definitions, publication of furnace ratings, and labeling for the information of the purchaser. The wording of a uniform guarantee and a list of the manufacturers, distributors, installers, testing laboratories, and user organizations which have officially accepted the standard are also included.

The pamphlet is available at 10 cents per copy from the Superintendent of Documents, Washington, D. C.

**Laboratory Equipment Takes Over for the Elements.**—Engines for airplanes of the future are constantly being tested by the Army Air Force to determine performance at different altitudes, temperatures and humidities. They are tested not only at altitudes as high as 40,000 feet, where temperatures drop to −67 deg. F. and where air pressure falls to 5.54 inches of mercury (absolute), but also at low altitudes over deserts, where the temperature soars to 120 deg. F., the relative humidity amounts to only 5 to 10 per cent., and the pressure may rise to over 29 inches of mercury. The engines undergoing test never leave the ground, but meet these extremes of atmospheric conditions in Army Air Force laboratories, where refrigeration equipment, electric heaters, and evacuating equipment are taking over for the elements. Such a laboratory will soon be put into operation in the United States. It will house several test chambers, all of which will be testing engines under different conditions. To accomplish this, air will be partly conditioned and then delivered to the various test chambers by a 250 horsepower blower. At each test chamber the air will be further conditioned to obtain the exact humidity, temperature, and pressure for the particular condition desired, and then delivered to the engine carburetors. If air of extremely low humidity is desired, the air will first be dropped in temperature to a point as low as −70 deg. F. to condense out the vapor, which will separate from the air in the form of snow or frost. The dried air will then be reheated to the proper temperature by General Electric finned Calrod heaters before being delivered to the carburetors. The total reheating power of this installation will be 1500 kw. The amount of moisture permitted to remain in the air will be regulated by the extent of chilling to which the air is subjected. Slight chilling drives out only a small amount of moisture. Steam will be mixed with the air to raise the humidity. Six General Electric motors will drive compressors which with the electric heaters will regulate the temperature of the air delivered to the carburetors. Evacuating equipment will produce any pressure between 29.92 inches of mercury at sea level and 5.54 inches at 40,000 feet. To simulate flying conditions completely, provision is made for cooling the gasoline to −10 deg. F. and for heating it to 80 deg. F.

R. H. O.

# THE FRANKLIN INSTITUTE.

## STATED MONTHLY MEETING, May 19, 1943.

The stated monthly meeting of The Franklin Institute, the last for the season, was held in the lecture hall at five o'clock, a departure from the regularly scheduled time.

After the playing of the National Anthem, the meeting was called to order by the President, Mr. Charles S. Redding.

He announced that the programme for Medal Day, which was presented at the last monthly meeting, was printed in the May number of the JOURNAL OF THE FRANKLIN INSTITUTE and that a detailed account of that meeting would be published in a later number of the JOURNAL. No objection being raised to this arrangement, he declared the account of the April meeting approved as printed.

Dr. Henry Butler Allen, Secretary, reported the following additions to membership:

| | |
|---|---|
| Sustaining | 3 |
| Active | 91 |
| Associate | 41 |
| Student | 15 |
| Total | 150 |

Mr. Redding announced that The Franklin Institute, having voted to elect Dr. Carl Van Doren to Honorary Membership, would present the Certificate at this meeting. He then gave a short sketch of Dr. Van Doren's life, pointing out that this noted editor and biographer had been awarded the Pulitzer Prize in 1939 for his book on Benjamin Franklin.

The formal presentation of the Certificate of Honorary Membership in The Franklin Institute was then made and accepted by Dr. Van Doren with expressions of appreciation for this valued honor.

Dr. Allen then presented to Dr. Van Doren photostatic copies of two hitherto unknown Franklin letters which had been unearthed through researches made by the National Franklin Committee, sponsored by the Institute.

Dr. Van Doren, the speaker for this meeting, discussed "Some New Facts About Franklin" and revealed also his own method of collecting his interesting material.

At six-ten the meeting adjourned to Franklin Hall where dinner was served. After dinner a round-table discussion took place, when the speaker answered several questions regarding Franklin's life, and members of the Institute spoke on various phases of Franklin's life which had claimed their interest.

The meeting ended at nine o'clock, with a rising vote of thanks to Dr. Van Doren for his generous contribution of time and interest in participating in this meeting to make Franklin better known.

## COMMITTEE ON SCIENCE AND THE ARTS.

*(Abstract of Proceedings of Stated Meeting held Wednesday, May 12, 1943.)*

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 12, 1943.

DR. JOSEPH S. HEPBURN *in the Chair.*

The following reports were presented for final action:

No. 2966: Work of J. S. Stokes in the Field of Packaging Machinery.

This report recommended the award of an Edward Longstreth Medal to J. Stogdell Stokes, of Huntingdon Valley, Pennsylvania, "In consideration of his development of paper box manufacturing machinery and his leadership of his organization in that field."

No. 3036: Hydroseal Pump.

This report recommended the award of an Edward Longstreth Medal to Frank B. Allen, of Narberth, Pennsylvania, "In consideration of the development of a pump which will successfully handle water or liquids containing abrasive materials, retain its initial efficiency and original discharge for considerable periods, with low maintenance costs, and the successful manufacture of this type of pump, giving satisfactory operation in many installations."

JOHN FRAZER,
*Secretary to Committee.*

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## LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and the reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from nine o'clock A.M. until five o'clock P.M., Thursdays from two until ten o'clock P.M. and Saturdays from nine until twelve o'clock noon.

### RECENT ADDITIONS.

#### AERONAUTICS.

BENNETT, D. C. T. The Complete Air Navigator. Fourth Edition. 1942.
BLACK, ARCHIBALD. The Story of Flying. Revised Edition. 1943.
FALK, KARL HANSSON. Aircraft Propeller Handbook. Revised Edition. 1943.
MACKINNON, H. D., JR. Aircraft Production Planning and Control. 1943.

#### AGRICULTURE.

ECKLES, CLARENCE HENRY, WILLES BARNES COMBS, AND HAROLD MACY. Milk and Milk Products. Third Edition. 1943.
MILLAR, C. E., AND L. M. TURK. Fundamentals of Soil Science. 1943.

### ARCHITECTURE AND BUILDING.

American Society of Heating and Ventilating Engineers. Guide. Volume 21. 1943

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## NOTES FROM THE BARTOL RESEARCH FOUNDATION.

### THE SPECULAR REFLECTION OF PLANE WAVE PULSES IN MEDIA OF CONTINUOUSLY VARIABLE REFRACTIVE PROPERTIES.*

BY

M. E. ROSE.†

The specular reflection of plane waves is treated by the wave theory method. For the case in which the velocity of wave propagation and the radiation impedance vary sufficiently slowly that multiple reflection may be neglected the reflection coefficient is first obtained for monochromatic waves incident on a set of $n$ plane parallel plates with otherwise arbitrary properties. The method may also be used to obtain the reflection coefficient in the case of large reflectivity but the results are exceedingly cumbersome for a large number of plates. The result for monochromatic waves is used to obtain the reflection coefficient for a plane wave pulse of arbitrary shape and duration. It is shown that the reflected wave train consists of a series of pulses of exactly the same form as the incident pulse with the amplitude of each reduced by the value of the appropriate Fresnel reflection coefficient. The interference effects are also represented. The sufficient condition for reproduction of the form of the incident pulse in the reflected wave is that the reflection coefficient for monochromatic waves be expressible as a sum of terms each of which is periodic in the frequency. In contrast to previous statements in the literature, periodicity of the coefficient itself is unnecessary. The results for the $n$ plate problem are generalized for both monochromatic waves and pulses for the case of stratified media having reflective properties which vary continuously in any manner. The special case of rectangularly shaped pulses is analyzed and application is made to the reflection in sea water and in the atmosphere.

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* Abstract of an article printed in full in *The Physical Review*, 63, 111 (1943).

† Now at Princeton University.

## CLOUD-CHAMBER TRACK OF A DECAYING MESOTRON.*

BY

R. P. SHUTT, S. DE BENEDETTI, AND T. H. JOHNSON.

In a previous letter [1] we described a track of what was evidently a negative mesotron at the end of its range in the gas of the cloud chamber. As with most of the other five or six published photographs showing mesotron stoppage in the

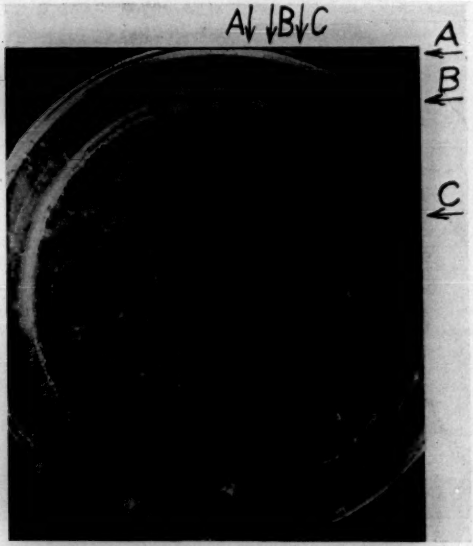


FIG. 1. Photograph of a mesotron disintegrating at *C*.

gas there was no evidence in our picture that the mesotron had disintegrated and until now a single photograph by E. J. Williams [2] has constituted the only indisputable cloud chamber evidence for mesotron decay. It is therefore of interest

* Reprinted from *The Physical Review*, **62**, 552 (1942).

[1] T. H. Johnson and R. P. Shutt, *Phys. Rev.*, **61**, 380 (1942).

[2] E. J. Williams and G. E. Roberts, *Nature*, **145**, 102 (1940).

to put on record another track of a mesotron disintegration recently obtained during first tests of a high pressure cloud chamber.

The details of the chamber will be described in another article and it will suffice to state that the chamber is cylindrical, 30 cm. in diameter and 10 cm. deep. It is designed

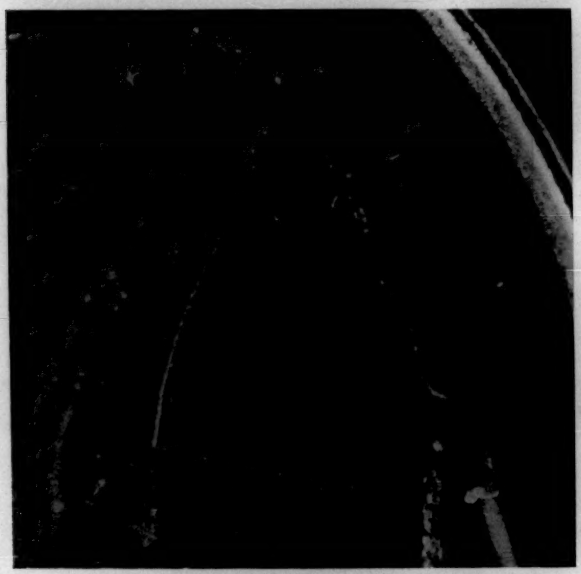

FIG. 2. Decaying mesotron of Fig. 1, enlarged.

to operate at an internal pressure of 200 atmospheres but the present photograph was obtained at a pressure of 70 atmospheres in a mixture of argon, *n*-propyl alcohol, and water vapor, without magnetic field or counter control. The expansion ratio decreases as the pressure is increased and at 70 atmospheres good tracks were obtained at a volume ratio of 1.05.

Figure 1 shows the entire field of the chamber, and Fig. 2 is an enlarged view of the track of particular interest. What is evidently a mesotron enters the chamber at $A$, at $B$ the track is definitely heavy, and at $C$ a conversion takes place in which the mass of the mesotron disappears and a lightly ionizing particle of high energy, presumably an electron, flies off at an angle of 85° with the direction of the primary mesotron track.

It is interesting to observe the apparent change in density of the mesotron track 5 cm. from its end at $B$. According to the Bethe-Bloch formula the residual range of a mesotron of mass 180 m is 6 cm. after the ion density in the track has become four times that of a ray of minimum ionization. Thus the apparent change in density occurring over a relatively short distance at this pressure is probably not an effect of uneven illumination. The curvature of the mesotron track is presumably caused by multiple scattering, and may be used for an estimate of the mass of the mesotron. According to Williams[3] $\rho_s/R \approx 1.3(m/Z)^{1/2}$, where $\rho_s$ is the average radius of curvature of the track over the first half of its range $R$, $m$ is the mass number of the particle, and $Z$ the atomic number of the scattering gas. Examination of the track shows that $\rho_s$ is between 2 and 6 times the residual range which places the mass of the particle between 40 and 400 electron masses. The lightly ionized track of the disintegration particle shows no deflection, and taking into account the high pressure of the gas we may conclude that its energy exceeds $3 \times 10^7$ e.v. whereas the density of the mesotron track indicates that its final energy was less than $10^6$ e.v.

In conclusion the picture seems to represent a slow particle of intermediate mass converted into a fast particle of small mass, the type of disintegration which has been postulated to account for certain anomalies in the absorption of cosmic rays in the atmosphere. Since this is the first track found in our experiments out of a total $2 \times 10^6$ cm. of mesotron tracks examined we have an estimate that the probability of disintegration is $5 \times 10^{-7}$ per cm. In our

[3] E. J. Williams, *Phys. Rev.*, **58**, 292 (1940).

high pressure chamber we can have an equivalent path length of 6000 cm. for each ray and with the long time of sensitivity realized we may have several rays in each photograph. Thus we may expect to find events of this type occurring once in something of the order of every one hundred photographs at maximum pressure.

Acknowledgment is made of the generous support given for the high pressure cloud-chamber project by the Carnegie Institution of Washington.

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## NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

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**Experimental Skin Sensitization with War Gases.**—RICHARD W. LINTON. The work we are reporting here grew out of an interest in the results published by Landsteiner and his collaborators in the *Journal of Experimental Medicine* between 1935 and 1941 as "Studies on the Sensitization of Animals with Simple Chemical Compounds." It seemed of interest to extend these methods to two substances which are of importance as war gases, namely phenarsazine chloride (DM) and chloroacetophenone (CN).

*Phenarsazine Chloride.*—A 1 per cent. solution of DM was prepared by dissolving the compound in a mixture of equal parts of dibutyl phthalate and olive oil. Each of a group of six guinea pigs was shaved on the side and five of them were treated by spreading a drop of the solution with a glass rod over an area about the size of a quarter; the corresponding area of the sixth pig was spread with a drop of the solvent mixture. This treatment was repeated at the same sites for 21 times at three day intervals. Two weeks after the last treatment, 0.1 ml. of a DM-albumin complex was injected intracutaneously at another site in each of the six animals. During the following 72 hours, no differences between the reactions of the experimental and control animals were observed, and it was concluded that no sensitization had taken place which could be detected by this method.

In a second experiment, a 2 per cent. solution of DM in ethyleneglycol monomethylether ("cellusolve") was used. In this solvent the DM does not cloud and precipitate as it does in the first solution. The spreading was carried out as before in a group of five guinea pigs and one control for 13 times at three day intervals, and two weeks after the last treatment a test was made by spreading a drop of the material at another site. No reaction was observed in any of the animals, nor

were any skin changes found during the course of treatment either in this group or in the previous one.

Using the same solution as in the first experiment, eight intracutaneous injections of 0.1 ml. each, were made into a group of five guinea pigs at three day intervals. A control animal received similar injections of solvent alone. No evidence of sensitization could be detected either by the subsequent spread of the DM solution on the skin or by the intracutaneous injection of a DM-albumin complex.

Numerous attempts were made to form a complex between DM and recrystallized egg albumin by shaking them together for long periods. It was assumed that the chlorine in the compound would be active enough to combine with the protein, as Landsteiner and Chase have shown to occur with picryl chloride (*Journal of Experimental Medicine*, **66**: 337, 1937) and Linton and Smith with the acid chloride of *p*-dimethylamino-*p'*-carboxyazobenzene (*Journal of the Franklin Institute*, **234**: 286, 1942). However, unequivocal proof that such a conjugate actually was formed under the conditions of our experiments could not be obtained.

*Chloroacetophenone.*—In contrast to the negative results with DM, the use of CN led to the production of a high degree of sensitization.

A solution of CN containing 10 mg. per ml. was made in equal parts of chloroform and olive oil. When kept tightly corked, the solution appeared to remain unchanged for a long time. Four groups of guinea pigs, each containing five or six animals, one of which served as a control, were shaved as before and treated with the solution (or the solvent alone in the case of the controls) at intervals of three days. After three or four treatments, all the experimental animals showed positive skin reactions at 24 hours, while the controls remained uniformly negative. The skin changes included reddening, swelling, scaling, loss of hair and in many cases the process went on to necrosis. The severity of the reactions appeared to reach a maximum after ten or twelve of the treatments, and from then on did not increase in severity, or actually were less marked, as the treatments continued.

Tests of sensitivity were made by rubbing the CN solution onto another part of the shaven skin. In 24 hours all the

experimental animals showed distinct reactions in the appearance of raised, red, well-defined areas which averaged 30 × 25 × 2 mm. in size. In one group which had received six treatments as against 10 or 15 for the other groups, the sites of reactions were slightly larger on the average than those in the other groups, which is in accordance with the above-mentioned increase of reactivity to a maximum. The control animals of the first three groups were negative at 24 hours but showed slight reactions at 48 hours. The changes consisted of minute, discrete, raised areas over the site of the test spread. In the last group, accordingly, the control animal was kept in a separate cage during the course of the sensitization, and when tested was completely negative, indicating the probability that the rubbing of CN solution from the experimental onto the control animals had been responsible for the slight positive reactions which had been previously obtained.

Before testing the last group of guinea pigs, the animals were bled, the serum separated and injected subcutaneously in 0.1 ml. amounts into a corresponding group of normal animals. Two hours later, CN in solvent was rubbed on the skin over the site of each injection. None of the experimental animals showed any more reaction than the control, which had received normal guinea pig serum followed by CN in solution.

CN shaken with egg albumin according to the methods of Landsteiner and Chase did not give a compound which elicited any sensitivity. The repeated subcutaneous injection of this material in guinea pigs did not lead to sensitization of the animals to subsequent skin tests with the CN dissolved in solvent.

*Summary.*—Guinea pigs have been rendered highly sensitive to chloroacetophenone (CN) by spreading the dissolved compound on their skins. Similar experiments with phenarsazine chloride (DM) did not lead to sensitization. The combination of these compounds with egg albumin did not lead to the formation of conjugates which elicited sensitivity, under the conditions of the experiments.

## BOOK REVIEWS.

PROTEINS, AMINO ACIDS AND PEPTIDES, by Edwin J. Cohn and John T. Edsall. 686 pages, illustrations and diagrams, 16 × 24 cms. New York, Reinhold Publishing Company, 1943. Price $13.50.

The structure and the functions of proteins are recognized generally as of fundamental importance in many fields. It has only been within the past twenty years that a great amount of progress has been made in such knowledge and today the structural pattern of protein molecules is still not known in detail. Certain general features of structure can, however, be regarded as established but the greatest problems in the structural chemistry of proteins are still unsolved. Their solution remains the goal of protein chemists.

Proteins are built of dipolar ions, the amino acids, and peptides. This book approaches the study of protein structure through these molecules and accordingly the simpler dipolar ions form the subject of Part I. It starts out by discussing spectroscopy and dipolar ionic structure where it is revealed that isoelectric amino acids and peptides exist as dipolar ions in solution. This comes from studies of Raman and infrared spectra. Then following is a review of the thermodynamic treatment of equilibrium, and the principles of the simpler electrical interactions affecting the thermodynamic properties of dipolar ions. A study is made of acid base equilibria which determine the relative amounts of all dipolar ions and ions carrying a net charge resulting from conversions by additions of acids or bases. Dielectric constants of the solutions of dipolar ions are examined and it is shown that their high magnitude is the direct consequence of their extraordinarily large electric moments. Other properties given attention include densities and heat capacities in water solutions; in fact several chapters are devoted to the consideration of solubility studies and of other methods of determining activity coefficients of amino acids and peptides.

Part II of the book is devoted to proteins. This problem is much more complex than that considered in Part I and for the most part no attempt is made to picture the structure of proteins in any detail. The treatment begins by considering the chemical evidence as to the mode of linkage of amino acids in the protein molecule and it is shown that the acids are bound together by peptide linkages. Proceeding further, X-ray diffraction studies are recorded leading to an approximation of molecular shape. Still further, the relative amounts of different amino acids which proteins yield on hydrolysis are recorded. Then the foundation as to the size of protein molecules is established through the subjects of osmotic pressures and of diffusion coefficients. These subjects include the contribution of Svedberg and his high-speed ultracentrifuge which development is touched upon and described. With regard to the shape of the protein molecule in solution, evidence is presented, derived from sedimentation and diffusion measurements, determinations of viscosity and double refraction flow, and dielectric constant measurements carried over a wide range of frequencies. Other sections are devoted to the subjects of the solubility of proteins—under which there is discussed the separation



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and classification of proteins—, and the interactions of proteins with ions and dipolar ions. Finally there is examined the theory of electrophoretic migration.

There is an appendix containing, in tables, dielectric increments of ellipsoidal molecules of different axial ratios. These are expressed as functions of the frequency of the applied field. A table of symbols, subject index, tabular index of amino acids and peptides, and tabular index of proteins complete the work.

This book is an American Chemical Society Monograph. Certain fundamental chapters have been prepared by different authors. These concern subjects in which they have made contributions. The book is authoritative and may be considered as an essential stepping stone for research in the physical chemistry of proteins.

R. H. OPPERMANN.

EMPIRICAL EQUATIONS AND NOMOGRAPHY, by Dale S. Davis. 200 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, Inc., 1943. Price $2.50.

Every engineer and laboratory worker is acquainted with the importance of empirical equations. To take a set of experimental data and make an expression of relationships between the variables is a task of no mean proportion, and more often than not the result will be a base upon which an entire project rests. Attention should therefore be given in all seriousness to doing this. The first part of this little book has for its purpose the explanation of the methods for the formation of empirical equations.

The author makes clear the picture at the very outset. He divides the determination of an equation into four steps, viz. the plotting of data; the identification of the plot with one of the familiar types of curves—straight line parabola, hyperbola, etc.; the rectification of the data or plotting of such functions of the variables that a straight line results; and the determination of the constants in the equation. There are followed through in some detail these four steps using the method of averages whereby the values of the rectifying functions, arranged in ascending or descending order with respect to at least one of the variables, are divided into two sets containing approximately the same number of pairs of values. The two sets are added separately, resulting in two equations, which are then solved simultaneously for the values of the constants in question. The procedure is facilitated with illustrations embodying actual data taken from engineering publications to which reference is made. Brief mention is made of the methods of selected points and least squares and a table compares the values for the three methods. Later there are presented several new techniques for two variable data and procedure for three variable correlations.

Part II of the book gives a detailed presentation of the theory and construction of the several industrially important types of nomographic charts and indicates briefly the structure of a few of the minor patterns. Illustrations are drawn from recent mechanical, civil and chemical engineering literature, and the problems given deal with charts now in actual daily use in the dairy, paper, textile, refrigeration, heavy chemical, petroleum, and other industries. Addition charts are covered as well as logarithmic, recurrent variable, non-logarithmic multiplication, combination, and line coordinate charts. At the end special attention is given engineering slide rules and slide rules for advertising.

There are many problems for exercise throughout the work as well as some 59 listed as problems of a miscellaneous nature. There is a bibliography of 190 references and an appendix of useful tabular data. A subject index completes the book.

The author has taken much care to present a useful text. By drawing heavily on practical illustrations and by adapting a good knowledge of the subject he has produced a means to the clarification and solution of many problems.

R. H. OPPERMANN.

THE CHEMICAL TECHNOLOGY OF PETROLEUM, by William A. Gruse and Donald R. Stevens. 733 pages, charts and diagrams, 16 × 23 cms. New York, McGraw-Hill Book Co., Inc., 1942. Price $7.50.

Seldom does a book dealing with a rapidly growing industry survive for any length of time. In fact in such cases the turn-over in books is excessive, and particularly with an industry such as petroleum this may be expected. How unusual it is then to note the demand for "Petroleum and Its Products" by Gruse over the years since it was published in 1928. Its curve of popularity would be interesting to see. Now, this book appears in its second edition. The chemical aspects of the industry, since 1928, have been described adequately by Americans in contrast to the previous foreign specialists. The modernizing of this book under the new title should be welcomed by everyone interested in a general chemical treatment.

The work starts out with the composition, classification, and description of crude petroleum, which lays the foundation for the discussion on the chemical constitution of petroleum, and group reactions of petroleum oils. A section devoted to physical properties is supplied by a collaborator, J. R. Bowman, Ph.D., and a discussion of the broad principles dealing with the contributions made by chemistry to the art of oil production is made by H. T. Kennedy, Ph.D. The present theories on the origin of petroleum are covered by dividing this topic into theories assuming an inorganic source for the oil and assuming an organic source. It is pointed out that the former hypotheses are generally rejected based on overwhelming evidence and considerably more space is devoted to the latter where here only tentative conclusions are reached.

Dr. Bowman then makes another contribution on distillation in which is featured a brief statement of some of the kinetic and thermodynamic ideas involved together with illuminating descriptions of separations of the components of volatile liquid mixture. Refining by chemical methods is next taken up, followed by a close consideration of the cracking of petroleum oils. Other collaborators—C. W. Montgomery, Ph.D., and J. H. McAteer, Sc.D.—present a review of the more useful thermodynamic relations, a summarization of the most recent data relating to hydrocarbons, and an application of the methods and data to the discussion of some important type reactions of interest to the industry. From here on the work is devoted to products. Gasoline and other motor fuels, kerosene, lubrication and petroleum lubricants, paraffin, amorphous waxes and petrolatum, fuel oils, and petroleum asphalts are taken up separately. Miscellaneous petroleum products and by-products conclude the work. An appendix by W. E. Hanson lists several of the more important physical properties of the principal hydrocarbons. In the back are author and subject indexes.

One of the Mellon Institute Technochemical Series, this edition should equal if not exceed the record of the first edition in demand and use. It is a dependable reference work and should go far in helping investigators and advanced students to gain entry into a field whose future is most bright.

R. H. OPPERMANN.

THE CHEMISTRY OF NATURAL COLORING MATTERS, by Fritz Mayer, Ph.D. 354 pages, diagrams, 15 × 23 cms. New York, Reinhold Publishing Corp., 1943. Price $3.00.

This book is an outgrowth of the third German edition of the second volume of "Chemie der organischen Farbstoffe" by Fritz Mayer, published in 1935. It has been revised to include recent findings up to the summer of 1941 to be in accord with the rapid progress in almost all branches of the subject. Among other things, the revision contains an account of some natural pigments which formerly escaped notice.

The book is divided into five general parts. The first part is devoted to carotenoids or polyene pigments. Here a close examination is made and good vision is obtained of the immensity and broadness of coverage existing not only here but throughout the book. The constitution of the pigments, their structure, their elaboration by nature and other important characteristics are all given consideration in the light of the many literature references used. Chapter II on diaroylmethane compounds is necessarily quite brief since only one natural pigment is known to belong to this group. It is curcumin which occurs in the roots and shoots of plants which grow in wild tropical Asia. The next section covers carbocyclic compounds and this is followed by heterocyclic compounds where considerable space is devoted to innumerable topics. The last part is headed *Compounds Containing Heterocyclic Nitrogen Atoms*. These pigments are classified as derivations of pyrimidine, of pyrrole, of indole, of pyridine and of pyrazine. A number of pigments of unknown constitution are also included. There are discussed the important pigments of the blood, bile, the green leaf and also the lyochromes (flavins). There is an author index and subject index in the back.

The book is the result of a tremendous amount of work, an intimate knowledge of the subject, and its progress. An American Chemical Society Monograph, it presents in English a survey of the chemical reaction known to exist among natural compounds which possess visible color, that has a definite place in research.

R. H. OPPERMANN.

FIRE PROTECTION ENGINEERING, by The National Fire Protection Association. 196 pages, illustrations, charts, and diagrams, 16 × 23 cms. Boston, National Fire Protection Association, 1943. Price $2.00.

This is a symposium of papers presented at a summer conference (June 1942) at the Massachusetts Institute of Technology. The conference was sponsored jointly by the School of Architecture and the Department of Building Engineering and Construction of the Institute and the National Fire Protection Association. It was attended by a representative group of fire protection engineers and fire department officials. Eighteen papers are included in the book by as many authors. The subject matter covers practically every phase of fire protection engineering. Fire losses, education, insurance, protection, hazards, control, city planning, legislation, and water supplies, are some of the subjects. An appendix

gives fire losses in the United States, 1900–1941. There is much valuable information in these papers from the standpoint of education generally as well as education through a formal course of instruction.

R. H. OPPERMANN.

COLLEGE PHYSICS, by Henry A. Perkins. Revised edition, 802 pages, illustrations, 16 × 23 cms. New York, Prentice-Hall, Inc., 1943. Price $4.50.

Since the first printing of the original edition of this book in March 1938, a great many students have become familiar with it. That edition went through six printings to September 1941. Books of this kind, up to the present emergency at least, have seldom been termed "best sellers" and it is indeed remarkable for this one to have so great a demand. The revised edition was undertaken, according to the author, in order to (1) add material dealing with some of the recent advances in physics and in their applications; (2) incorporate certain changes for the purpose of uniformity; (3) take advantage of many suggestions and criticisms by those who have used the book; (4) add some 345 problems without answers, corresponding to analogous problems with answers in the original text.

There are six different parts to the book starting with mechanics and followed by heat, wave motion and sound, light, electricity and magnetism, and corpuscular physics. The treatment of each is thorough, and somewhat rigorous especially in the early stages of each of the subjects. Later, when the groundwork has been laid reference is made to practical, everyday objects and situations and the work becomes more practical. The coverage is rather detailed. Only very simple algebra and trigonometry, including vectorial representation, are used. While little space is wasted for other than strictly the subject at hand, a good effort is made to make it impressive regardless of the space taken within reasonable limitations. The result is a book featuring clarity.

At the end of each chapter there are suggestions for supplementary reading and problems of two kinds—one set with answers and the other without. Both sets of problems reflect care in selection of type and composition. The number of them verifies the text in indicating a conviction that the study of physics, to be successful, must involve the solution of many problems. They also add value to the book for home study. A book on college physics also must be so constructed that it offers continuity in the study of physics from the elements to the intermediate stage. This book has many good qualities to do this. The book in its present form is a good modern text of college grade that provides basic knowledge for the further study of scientific and engineering subjects.

R. H. OPPERMANN.

METALLOGRAPHY, by Cecil H. Desch, D.Sc. Ph.D. F.R.S. 408 pages, illustrations, 16 × 23 cms. New York, Longmans, Green and Company, Inc., 1942. Price $8.00.

Practical metallurgy has contributed as much perhaps as any other activity in technological and scientific progress. It has done this with far less spectacular exhibitions and the situation at present holds as much promise as other better advertised fields. The needs of metallurgy have been the reasons for metallographic investigations. Such work has provided a means of identifying the relations in metallic and non-metallic systems and metallography is a very important auxiliary in the control and inspection of products of the metallurgical industries. As a distinct subject, its future is bright and the rate of its progress is so high that

a text must necessarily lag behind actual developments to a more than usual extent. The rate of progress is best illustrated by the fact that a reprint of the fourth edition of the book at hand came out in August 1939 and the present edition is the fifth, it having appeared in January 1942.

The fifth edition of any book on a subject of this kind is a testimonial to its worth. This work proceeds along lines generally similar to the previous editions. The basic methods of investigation in the study of metallography are recognized and treatment is given of the thermal, microscopical, and X-ray methods in quite some detail. A general outline of the crystallization of metals is given and the mechanism of growth is examined. Other accounts specifically relate to undercooling and the metastable and labile status; the possibility of diffusion in the solid state in which an example is given taken from the Bakerian lecture of Roberts-Austen; the physical properties of alloys such as density, electrical conductivity, electrolytic potential, and the chemical behavior of alloys; the structure of cold worked metals; the construction of the equilibrium diagram; the constitution of alloys and the nature of intermetallic compounds; transformations in the solid state. A separate treatment is given of the metallography of iron and steel with stress being laid on the equilibrium of iron and carbon with lesser emphasis on the modification brought about by the introduction of other elements into the system.

The work is thorough along the lines chosen. The many references given enable a further study of pertinent parts. There are no tables of known systems, dependence being laid on the use of outside indexes easily available.

R. H. OPPERMANN.

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*Potash in North America*, J. W. Turrentine, Ph.D. 186 pages, illustrations, 16 × 23 cms. New York, Reinhold Publishing Co. 1943. Price $3.50.

*Applied Electronics*, by Members of the Staff of the Department of Electrical Engineering, Massachusetts Institute of Technology. 772 pages, charts and diagrams, 15 × 23 cms. New York, John Wiley & Sons, Inc. 1943. Price $6.50.

*Vector and Tensor Analysis*, by Homer V. Craig. 434 pages, diagrams, 16 × 23 cms. New York, McGraw-Hill Book Company. 1943. Price $3.50.

*Dictionary of Science and Technology, in English, French, German, Spanish*, by Maxim Newmark, Ph.D. 386 pages, 16 × 23 cms. New York, Philosophical Library, Inc. 1943. Price $6.00.

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*Magnetic Circuits and Transformers*, by Members of the Staff of the Massachusetts Institute of Technology. 718 pages, charts and illustrations, 16 × 23 cms. New York, John Wiley & Sons, Inc. 1943. Price $6.50.

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## CURRENT TOPICS.

**Methyl Bromide is Versatile.**—Methyl bromide is one of the most effective and versatile fumigants now available, the U. S. Department of Agriculture reports. Tests have shown that it is useful against all forms and stages of insect life infesting plant material, and apparently safe for fumigating food products. Within the last five years quarantine workers, nurserymen, and various food storage industries have applied these findings to a wide variety of insect problems. The gas penetrates well and reaches sheltered pests like leaf miners, borers, mites, and other internal feeders on plants, and also burrowing rodents. In solution with water, methyl bromide is a good fumigant for small areas of soil infested by white-fringed beetles and other pests that pass some stages underground. Some of the many warehoused food products saved by methyl bromide fumigation are stored grains—rice, wheat, flour and feeds—dried fruits, nut meats, dry beans, coffee beans, chicory, and spices. In quarantine work methyl bromide is authorized for treatment of practically all imported plant material and for imported green-pod vegetables, cipollini bulbs, and chestnuts. It is also used for nursery and greenhouse plants and fruits and vegetables likely to be infested with the Japanese beetle; for potato tuber moth; sweetpotato weevil, azalea leaf roller and miner; European corn borer; gladiolus thrips; and many other insect pests. Most living plant material is not affected by ordinary dosages of methyl bromide, although overdoses might injure any plants. As a gas, methyl bromide is injurious to all forms of animal life. Hence operators in fumigating work need training, and must take certain precautions in handling the gas. Chief protection is afforded by wearing an approved gas mask, and testing the concentration by means of a halide detector before entering a fumigating vault or chamber handling fumigated material.

R. H. O.

**Wood Manhole Lids Fill Wartime Need.**—Wooden manhole covers are helping war booming communities solve a problem posed when the War Production Board banned the ordinary iron variety. First tried by Los Angeles County, California, the wooden manhole covers built of solid timbers specially treated with a salt preservative to resist termites and decay, have proved highly successful. The WPB order was designed to save 500 pounds of critical metal

which normally goes into each cast iron installation—250 pounds in the lid and 250 pounds in the frame. Some cities already have experienced delays in the installation of sewage facilities for war housing projects due to a shortage of metal manhole covers. The Los Angeles County surveyor's office developed the wooden substitutes and Surveyor Alfred Jones asserts the idea is suitable for city and county use throughout the nation. These wooden lids may be constructed in either circular or hexogonal design by laminating short lengths of plank. They may be built in local wood-working shops without extensive fabricating equipment. Tests show that the pressure-treated wooden covers have ample strength and durability.

R. H. O.

**Sanguinarine Rivals Colchicine.**—The drug, sanguinarine, may prove a rival to colchicine, the chemical that has stirred scientific interest and popular curiosity by its effects in modifying plant inheritance. Treating seeds or seedlings with colchicine has caused a doubling of the chromosomes in some cases, thus creating new, and sometimes superior, strains. Thomas M. Little, of the U. S. Department of Agriculture, recently reported similar results from preliminary experiments with a weak solution of sanguinarine applied to snapdragons. In this limited test the new treatment caused even more plant modifications than a parallel application of colchicine, and was somewhat less injurious to the treated plant material. Colchicine is extracted from the juices of the plant *Colchicum*, or meadow saffron. Sanguinarine gets its name from *Sanguinaria canadensis*, the scientific name of the common Eastern wildflower called bloodroot because its juice is blood red. Indians used the juice as medicine, and it is now sometimes used in cough medicines and in treatment of certain skin diseases. The drug is known chemically as an alkaloid and is somewhat poisonous, though not so much as colchicine and strychnine. Dr. Glenn A. Greathouse, of the Department, found extremely weak solutions of sanguinarine effective in preventing the growth of the destructive root rot fungus, and his experiments led Little to the successful treatment of snapdragons for modification of their inheritance.

R. H. O.

**An Electronic Circuit for Studying Hunting.**—Manuel J. De Lerno and Robert T. Basnett, in Electrical Engineering, Vol. 61, No. 12, describe an ingenious method for the determination of the hunting characteristics of synchronous machinery that involves simple apparatus, not difficult to obtain or set up, and is readily

adaptable to equipment already in service. The fundamental idea is that a mirror, placed on the shaft of a synchronous machine so as to reflect a beam of light from a lamp into a photoelectric cell every time the rotor of the machine is in a particular position, will cause the cell to receive flashes of light at equal time intervals. However, if hunting occurs, the time between successive flashes will vary: As the speed increases, the time decreases; conversely, as the speed decreases, the time increases. If a capacitor is arranged so as to begin charging at a constant rate at the same point on every wave of the potential applied across its terminals, and this capacitor is connected to the photoelectric cell in such a manner that every time the cell receives a flash of light the capacitor is discharged, then the charge on the capacitor varies as the length of time between flashes varies. The device that was evolved is made up of four distinct electrical and two mechanical units. The two mechanical aids consist of a lamp for producing a focused beam of light, and a metal band to which are attached several small mirrors, one for each pair of poles of the machine under test. The fundamental unit of the electrical system is a circuit for charging a capacitor at a constant rate, accomplished by connecting a vacuum pentode in series with a gaseous tetrode and a capacitor. The grid potential of the tetrode is governed by the second electrical unit—a gaseous photoelectric cell and its amplifier. The third unit is the power supply consisting of a type 80 tube and a filter section, and the fourth is a cathode-ray oscilloscope or a magnetic oscillograph. When the synchronous machine is operating at constant load so that no hunting occurs a triangular wave appears on the oscilloscope provided the proper sweep frequency is chosen. A change in load on the machine causes an increase or decrease in height of the triangular wave which is proportional to the shift in phase between the rotor and the applied voltage. During the period of instability, or hunting, the amplitude of the wave is constantly varying, and by means of refinements in the instrument a record of the hunting cycle is obtained.

R. H. O.

**Water Resources 1942.**—According to a report by the Geological Survey, United States Department of the Interior, in collaboration with the Dominion Water and Power Bureau, the past 12-month period has been the second consecutive year of reverse contrasts as between the central west and east. The year too, has been notable for its floods, most of which were of limited areal extent, but severe; many were record breaking. Artificial storage was well maintained and generally favorable ground-water conditions prevailed over the

United States. The wide-spread rise of ground-water levels during 1942 in the Central States indicates that ground water storage in that area, which was so seriously depleted, is now the greatest in recent years. The inexplicable tendency of wet periods and dry periods to persist or to change slowly is demonstrated by the fact that the 12 months ended September 30, 1942, like the water year ended September 30, 1941, was again decidedly "upside down" with 350,000 square miles in the Eastern States having a runoff of only 69 per cent. of normal (6th driest year in 30 years), whereas in the central West 625,000 square miles had an average runoff of 240 per cent. of normal and for that area as a whole was, in terms of stream flow, the wettest year of record by a very considerable margin. Areas in central Western States that were labeled "the dust bowl" during the drought of the 1930's, experienced a total runoff for 24 months ended September 30, 1942, averaging 200 to 300 per cent. of normal, indicating that the designation of the thirties is not applicable at present. At index stations in the north Central States monthly discharge during more than half the year exceeded previous monthly maxima and the total runoff for the year exceeded normals which embraced runoff records during the exceptionally wet years at the turn of the century. Although the drought persisted in the East throughout the greater part of the year and the runoff for the year averaged below normal, copious and well timed rains during the late summer and early fall produced normal or above normal stages in the rivers and caused considerable ground-water recharge.

R. H. O.

**Plastic Petroleum for Anti-Aircraft Guns.**—A remarkable plastic petroleum that is being extensively used to lubricate the magazines of marine anti-aircraft guns has been developed by the Gulf Oil Corporation, it was announced recently. It is put in the magazines of the Oerlikon gun, a rapid fire 20 mm. gun first developed abroad but improved and first mass produced here for the U. S. Navy. In addition to this use, these compounds enter other important military service. The gun magazine looks like a large box. Inside is a strip of cartridges and a large spring that looks like a watch spring magnified a thousand times. To operate the rapid firing anti-aircraft gun, after the magazine is attached, the spring is wound up with a geared crank. During firing, the spring tension keeps revolving the magazine to feed cartridges to the gun as rapidly as required. When the magazine has been emptied it is removed and a full one installed and wound up. The springs must be lubricated in such a manner that they will function smoothly in freezing

Alaskan or Icelandic waters or on the hot China or South Pacific seas. They must be protected from rust and corrosion. Anti-aircraft guns installed on ships often are drenched with salt water when waves pass over the decks and this lubricant must be of a sort that will not permit the salt water actually to touch the steel. Plastic petroleum compounds answer these requirements and have gained a reputation for unique and somewhat mysterious properties. The discovery of the existence of these properties was made by Gulf technologists while tackling the problem of lubricating business machines. First a plastic material which would stick on the metal parts at high operating temperatures was developed. This was done by mixing high melting point waxes and oil-giving products which tended to set to a hard consistency on standing. Gulf technologists then found that the addition of an undisclosed ingredient to the wax-oil mixture reduced the tendency of the plastic petroleum to set.

R. H. O.

**Conservation of Lumber in Boat Building.**—Carl A. Rishell of the National Lumber Manufacturers Association recently discussed the part of the lumber industry in wood boat construction. He pointed out that it should be remembered that to cut and deliver bending oak and ship timbers takes considerable time. It has been estimated that for oak timbers only one tree out of 200 is suitable for boat building. The further development of laminating boat parts will permit the other 199 oak trees to be used to a large extent for boat building. In fact if laminating is used on a large scale, it will be the most important single factor in maintaining an unfailing supply of material. It has been made possible for boat construction by the new glues on the market for the past three or four years. About two and one-half years ago the co-operation of the Navy in a program of research on lamination was secured. Experiments have been conducted by the Navy department with the advice and assistance of the Forrest Products Laboratory. More recently interest of the Coast Guard was created and a glued laminated keel was built with the stem and horn timber attached. It is believed that the success of the various experiments conducted in glued laminated keels and frames will encourage the government and the lumber industry to construct laminating plants where boat keels, sterns, frames, and other parts can be laminated in any size required. Lamination will allow the taking of long lumber of low grade, cutting it up to secure clear material, and glueing the short pieces together to provide structural parts which are stronger than the solid materials used in the past. In addition it will be possible to procure timbers which have been dried to the proper moisture

content. At the moment there are no large, solid timbers available in low moisture content. Large timbers of the sizes needed for ship construction may take two to five years to dry. With laminating, it is proposed to cut all the lumber into inch thicknesses, dry it in dry kilns to the proper moisture content, laminate it into timber of the proper size and ship it in perfect condition in all respects.

R. H. O.

**Antirust Insurance for Airplane Engines.** A. M. HOFFMAN. (*Compressed Air Magazine*, Vol. 48, No. 1.) Moisture is an enemy which can do great damage to engines between the time of their test runs and delivery necessitating considerable delay to fit them for service. To obviate such delay there has been worked out an engine preservation program that had its beginning in 1939 when Esso Marketers began laboratory work on a corrosion preventive that would neutralize the acidic products resulting from the combustion of high antiknock airplane fuels; provide protection against high humidity during protracted storage; have a minimum effect on the various metals and materials that enter into the construction of an engine; have no harmful effect on the lubricating oil; and be nonoxidizing and nongumming in storage. Upon the discovery of a preventive that seemed to meet all requirements, the armed services, engine builders, and the Aeronautical Materials Standards Subcommittee of the SAE undertook to investigate the matter, with the result that a 3 point program of engine protection was adopted. This involves the use of rust-prevention materials; placing a dehydrating agent in and around each engine; and wrapping the engine in a moisture proof container. Cylinders are coated by introducing a quantity of the solution into the fuel-induction system just before the engine is stopped and the moisture has a chance to condense. It is said that one pint of Rust-Ban 604 for every 1000 cubic inches of piston displacement is sufficient to cover the cylinder walls, pistons, and valve heads. Interior surfaces are treated in different ways, one of which is by using the preventive as a lubricant for the last 15 to 30 minutes of the test run. The exterior of the engine is protected by means of silica gel, a pure silicon oxide that can absorb as much as 40% of its weight in moisture. This material is chemically treated so that it gradually changes in color from blue to pink under the influence of moisture, the shading indicating the percentage of moisture. It is packed in cotton bags, and one of these is suspended from each cylinder. In addition, a transparent plastic plug with small perforations in the base and containing silica gel is screwed into each spark plug opening to take care of

moisture inside the cylinders. So safeguarded, the engine is encased in a Pliofilm bag that is impervious to moisture.

R. H. O.

**New Electronic Variable Speed Drive.** How a General Electric electronic variable speed drive called the Thy-mo-trol not only saves time and material but also helps to increase the amount and kinds of work that can be done on various machines, was described recently at a meeting of the American Society of Tool Engineers at Boston, Mass., by S. D. FENDLEY, G.E. electronic engineer. Mr. Fendley preluded his talk with the statement that while the use of electronic tubes for controlling motors dates back more than a decade, these applications were used primarily to control speed. All other functions were obtained through the use of various combinations of conventional equipment. Using a special demonstration unit, the speaker then showed that the Thy-mo-trol system provides features that are not inherent in conventional drives. It employs electronic tubes to convert a-c to controlled d-c, and provides variable armature and field voltages for operating a d-c motor from an a-c power line. The equipment not only provides wide-range speed control but performs the additional functions of starting, accelerating, protecting, and stopping motors with ratings up to 5 hp and 230 volts. Mr. Fendley then explained in detail the operation of the Thy-mo-trol drive and described how it is being used on various types of machines. He mentioned instances where the wide speed range of this system has enabled machines to produce more kinds of work and in altogether greater volume. He mentioned another instance where the ability of the equipment to maintain full load torque, even when operated at low speed, made it possible for a machine builder to simplify materially the headstock of his machine. The adjustable limit feature of the drive makes it possible to limit the motor torque to a predetermined safe value. Overloading the machine beyond safe limits is thereby prevented, as is the possibility of subjecting the gears and other parts to undue shock during acceleration. Too, if a serious overload occurs an overload relay stops the motor, and when the stop button is pushed, dynamic braking quickly and smoothly stops the motor, thus preventing loss of time due to coasting.

R. H. O.

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